For the system shown in Fig. 3.49, \( x \) and \( y \) denote, respectively, the absolute displacements of the mass \( m \) and the end \( Q \) of the dashpot \( c_1 \). (a) Derive the equation of motion of the mass \( m \), (b) find the steady-state displacement of the mass \( m \), and (c) find the force transmitted to the support at \( P \), when the end \( Q \) is subjected to the harmonic motion \( y(t) = Y \cos \omega t \).

**FIGURE 3.49**
Solution for the problem of Figure (3.49)

(a) Equation of motion of mass:

\[ m \ddot{x} = c_1 (\dot{y} - \dot{x}) - c_2 \dot{x} - \kappa_2 x \]

\[ x(t) \]

\[ \kappa_2 x \]

\[ m \]

\[ c_1 (\dot{y} - \dot{x}) \]

i.e.,

\[ m \ddot{x} + (c_1 + c_2) \dot{x} + \kappa_2 x = c_1 \dot{y} = -c_1 \omega \gamma \sin \omega t \]

(b) \[ x_p(t) = \frac{-\left(\frac{c_1 \omega \gamma}{\kappa_2}\right)}{\sqrt{(1-r^2)^2 + (2\gamma r)^2}} \sin(\omega t - \phi) \]

where \( r = \omega/\omega_n \), \( \gamma = (c_1 + c_2) \omega/(2r \kappa) \) and \( \phi = \tan^{-1}\left(\frac{2\gamma r}{1-r^2}\right) \).

(c) steady-state force transmitted to point P:

\[ = \kappa_2 x_p + c_2 \dot{x}_p \]

\[ = \frac{-\left(\frac{c_1 \omega \gamma}{\kappa_2}\right)}{\sqrt{(1-r^2)^2 + (2\gamma r)^2}} \left\{ \sin(\omega t - \phi) + \frac{c_2 \omega}{\kappa_2} \cos(\omega t - \phi) \right\} \]
Find the steady state response amplitude and the phase angle of the harmonically excited system shown in the figure.
\(m\ddot{x} = -k_1x - cx + f_0 \sin(wt) - T\)

\(T = -m\ddot{x} - k_1x - cx + f_0 \sin(wt)\)

\[-k_2 r^2 \dot{\theta} + T * 2r = J_o \dot{\theta}\]

\[-k_2 r^2 \dot{\theta} + (-m\ddot{x} - k_1x - cx + f_0 \sin(wt)) * 2r = J_o \dot{\theta}\]

Equation of motion for rotation of pulley about \(O:\)

\[-k_2 (\theta r) r - J_o \dot{\theta} - k_1 x (2r) - c \dot{x} (2r) + F_0 \sin \omega t (2r) - m \dddot{x} (2r) = 0\]  
where \(\dot{\theta} = \dot{x}/(2r)\)  
Equation (1) can be rearranged as:

\[\left(\frac{J_o}{2r} + 2m r\right) \ddot{x} + 2c r \dddot{x} + \left(2k_1 r + \frac{1}{2} k_2 r\right) x = 2r F_0 \sin \omega t\]  
(2)

For given data, Eq. (2) becomes

\[11 \dddot{x} + 50 \dddot{x} + 112.5 x = 5 \sin 20 t\]  
(3)

Steady state response is given by Eq. (3.25):

\[x_p(t) = X \cos (\omega t - \phi)\]

where \(X = \frac{5}{\left[\left(112.5 - 11 (20^2)\right)^2 + \left(50 (20)\right)^2\right]^{1/2}} = 0.001136\) m

\[\phi = \tan^{-1}\left(\frac{50 (20)}{112.5 - 11 (20^2)}\right) = -0.2291\ \text{rad} = -13.1287^\circ\]
One of the tail rotor blades of a helicopter has an unbalanced mass of \( m = 0.5 \text{ kg} \) at a distance of \( e = 0.15 \text{ m} \) from the axis of rotation, as shown in Fig. 3.29. The tail section has a length of 4 m, a mass of 240 kg, a flexural stiffness \( (EI) \) of 2.5 MN \(-\) m\(^2\), and a damping ratio of 0.15. The mass of the tail rotor blades, including their drive system, is 20 kg. Determine the forced response of the tail section when the blades rotate at 1500 rpm.
\[ k = \text{spring constant of cantilever beam} \]
\[ = \frac{3EI}{l^3} = \frac{3 \times (2.5 \times 10^6)}{4^3} \]
\[ = 0.1172 \times 10^6 \text{ N/m} \]

\[ \omega_n = \sqrt{\frac{k}{m_1 + 0.25 m_b}} = \sqrt{\frac{0.1172 \times 10^6}{20 + 0.25 \times 240}} = 38.2753 \text{ rad/sec} \]

\[ \omega = 2\pi (1500)/60 = 157.08 \text{ rad/sec} \]

\[ r = \frac{\omega}{\omega_n} = 157.08/38.2753 = 4.1040 \quad , \quad r^2 = 16.8428 \]

Forced response is given by Eq. (3.79):
\[ x_p(t) = X \sin(\omega t - \phi) \]

where
\[ X = \frac{m e}{m_1} \frac{r^2}{\sqrt{(1-r^2)^2 + (2 \zeta r)^2}} \]
\[ = \frac{(0.5)(0.15)}{20} \frac{16.8428}{\sqrt{(1-16.8428)^2 + (2 \times 0.15 \times 4.1040)^2}} \]
\[ = 3.9747 \times 10^{-3} \quad m = 3.9747 \text{ mm} \]