



Mechanical Vibrations

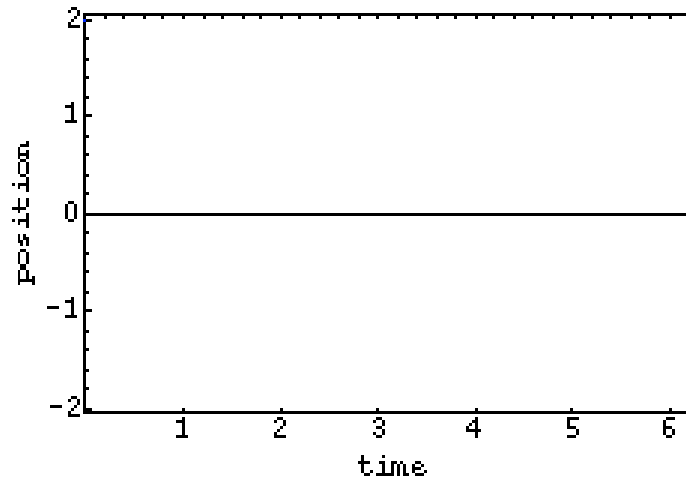
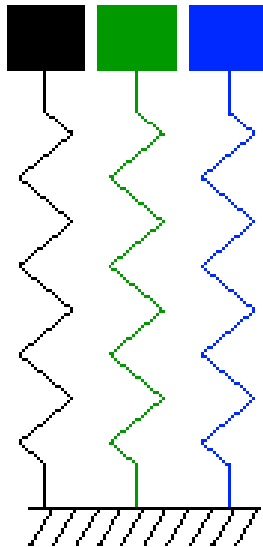
1



Introduction to Mechanical Vibrations

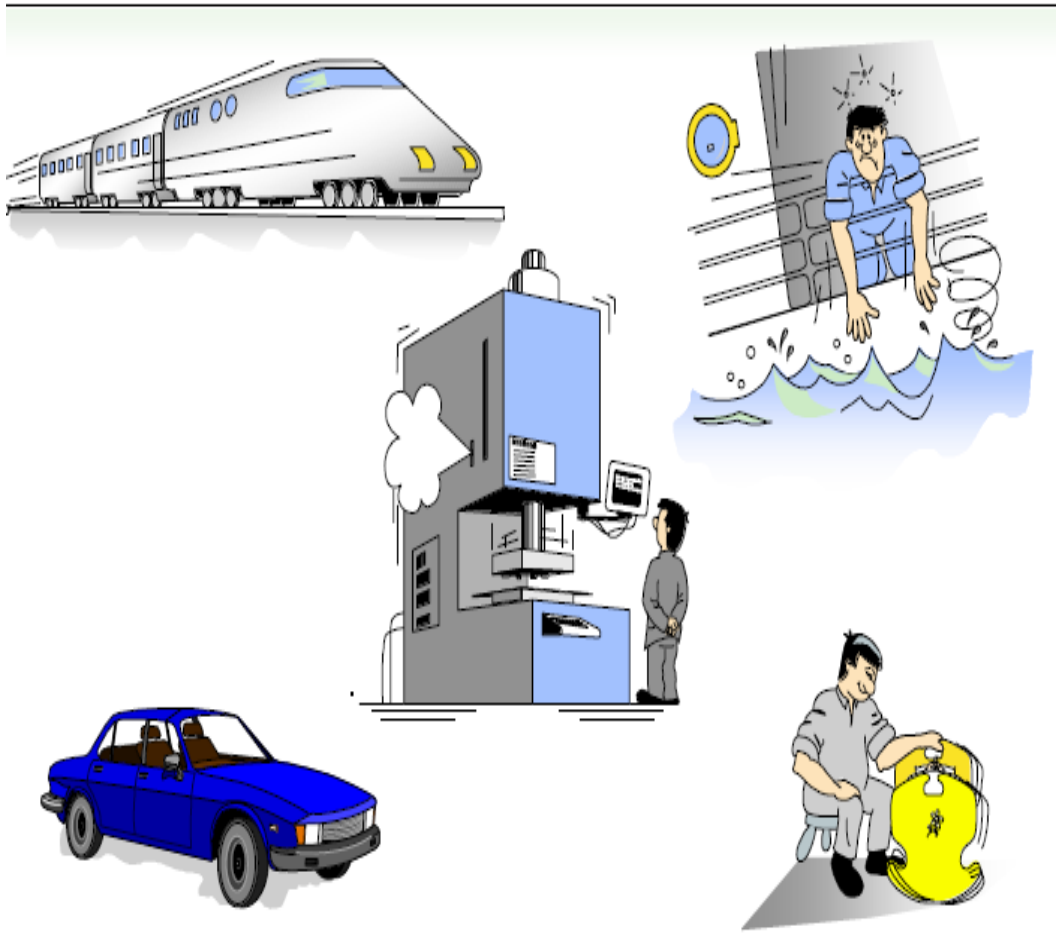
What is vibration?

- Vibrations are oscillations of a system about an equilibrium position.



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Vibration...



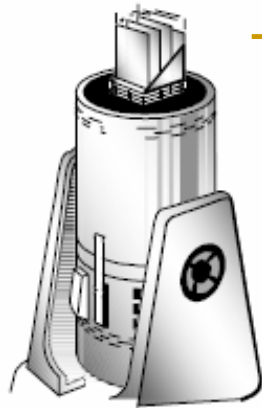
It is also an
everyday
phenomenon we
meet on
everyday life

Vibration ...

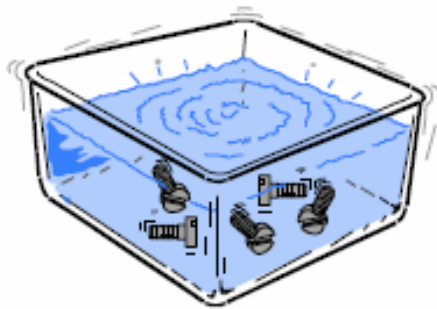
Useful Vibration



Compressor



Testing



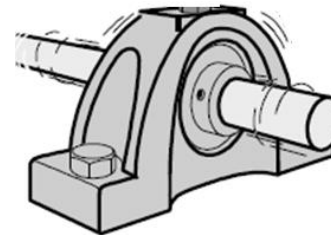
Ultrasonic
cleaning

Harmful vibration



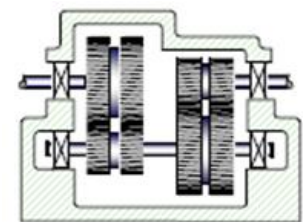
Noise

Destruction



Wear

Fatigue



Importance of the Study of Vibration

■ Why study vibration?

- ✓ Vibrations can lead to excessive deflections and failure on the machines and structures
- ✓ To reduce vibration through proper design of machines and their mountings
- ✓ To utilize profitably in several consumer and industrial applications
- ✓ To improve the efficiency of certain machining, casting, forging & welding processes
- ✓ To stimulate earthquakes for geological research and conduct studies in design of nuclear reactors

Modelling of vibrating systems

- Analytical Method (Continues Method)

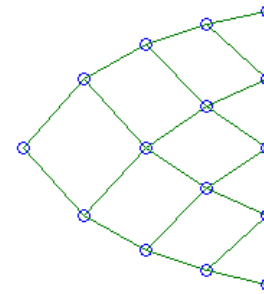
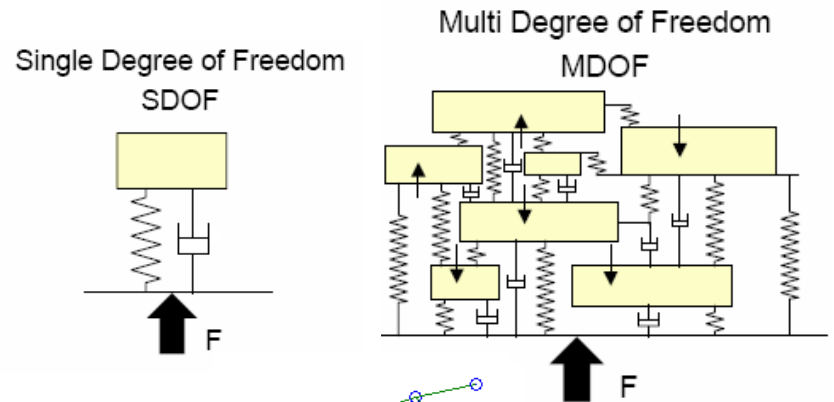
$$\frac{\partial}{\partial x} \left(EA \frac{\partial u}{\partial x} \right) + f = \rho \frac{\partial^2 u}{\partial t^2}$$

- Approximate Method

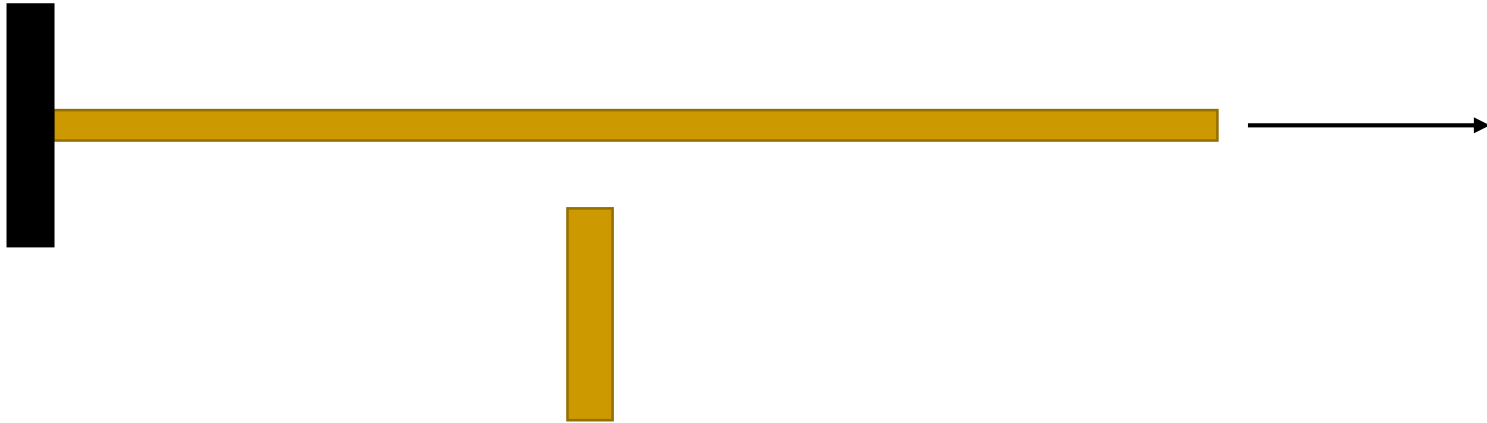
Lumped (Rigid) Modelling

Numerical Modelling (FEM,BEM)

- Experimental Method (Modal Testing)



Analytical Method (Continues Method)



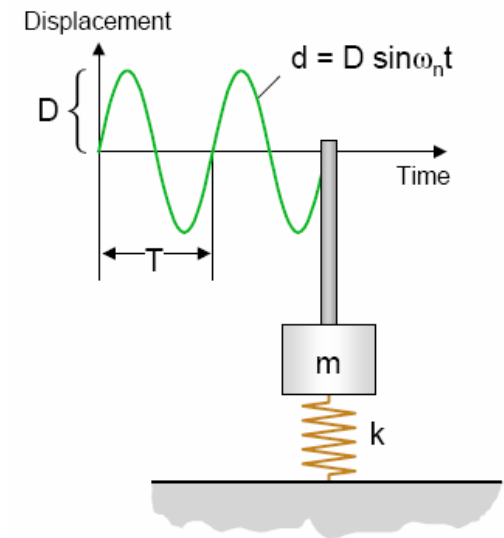
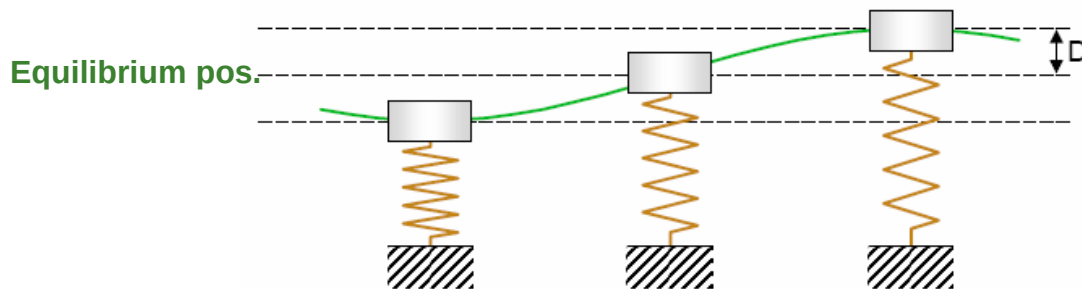
$$p + dp - p + f dx = ma$$

$$dp + f dx = \rho dx \frac{\partial^2 u}{\partial t^2}, \quad dp = \frac{\partial p}{\partial x} dx \rightarrow \frac{\partial p}{\partial x} + f = \rho \frac{\partial^2 u}{\partial t^2}$$

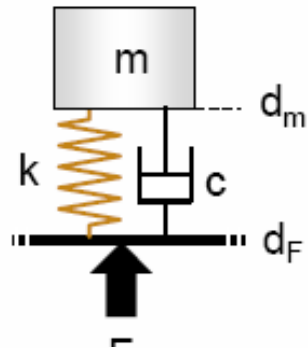
$$p = \sigma A = E \varepsilon A = E \frac{\partial u}{\partial x} A \rightarrow \frac{\partial}{\partial x} (EA \frac{\partial u}{\partial x}) + f = \rho \frac{\partial^2 u}{\partial t^2}$$

Free vibration

- When a system is initially disturbed by a displacement, velocity or acceleration, the system begins to vibrate with a constant amplitude and frequency depend on its stiffness and mass.
- This frequency is called as **natural frequency**, and the form of the vibration is called as **mode shapes**

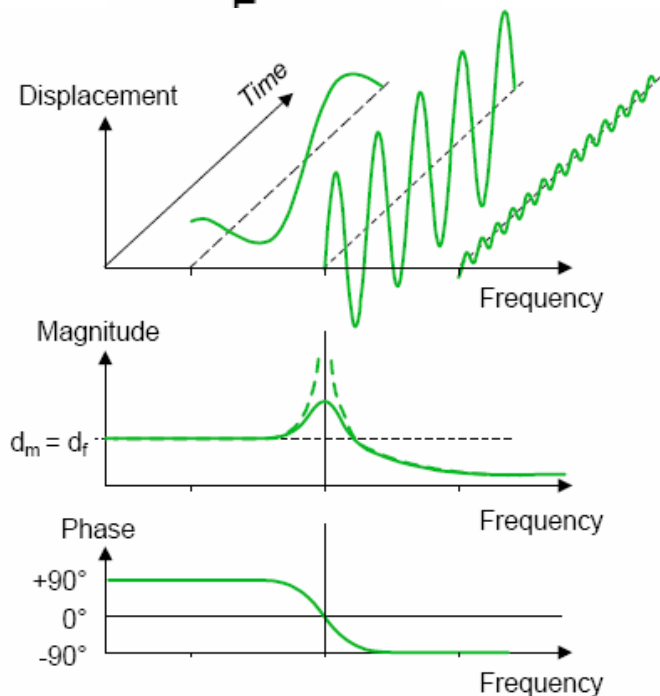


Forced Vibration



If an external force applied to a system, the system will follow the force with the same frequency.

However, when the force frequency is increased to the system's natural frequency, amplitudes will dangerously increase in this region. This phenomenon called as **“Resonance”**

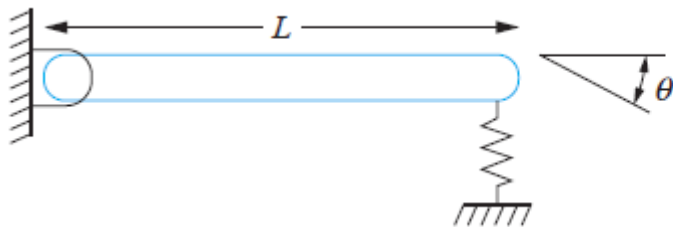


Degree of Freedom (DOF)

- Mathematical modeling of a physical system requires the selection of a set of variables that describes the behavior of the system.
- The number of *degrees of freedom* for a system is the number of kinematically independent variables necessary to completely describe the motion of every particle in the system

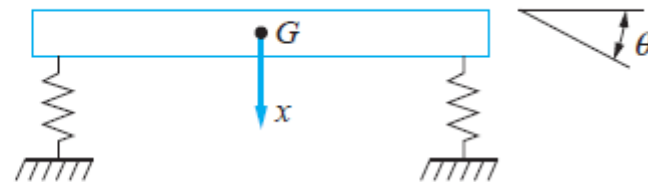
DOF=1

Single degree of freedom (SDOF)



DOF=2

Multi degree of freedom (MDOF)

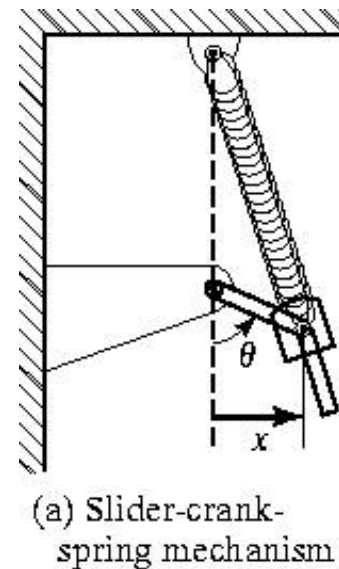
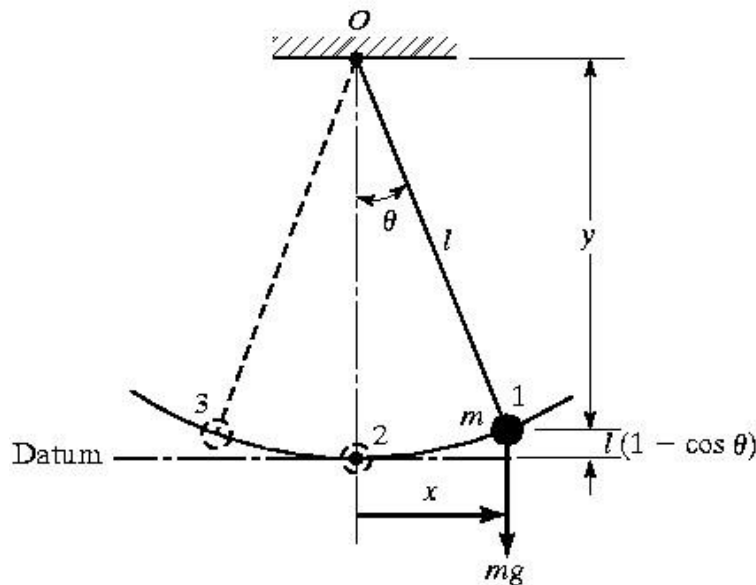


بودجه‌بندی درس

شماره هفته آموزشی	مبحث	توضیحات
۱	آشنایی با سرفصل درس، منابع درس و نحوه ارزشیابی - بیان مقدمه و مرور مبانی ریاضی مرتبط با درس	
۲	مروری بر دینامیک اجسام صلب و بیان مفاهیم اولیه ارتعاشات، درجه آزادی، فرکانس، پدیده تشدید،	
۳	ارتعاشات آزاد یک درجه آزادی - ارتعاش نامیرا، استخراج معادله به روش معادلات حاکم	
۴	ارتعاشات آزاد یک درجه آزادی - حل معادله ارتعاش آزاد تحت شرایط اولیه	
۵	ارتعاشات آزاد یک درجه آزادی - معرفی انواع میراگر، استخراج و حل معادله حاکم بر حرکت با وجود میراگر ویسکوز	
۶	ارتعاشات آزاد یک درجه آزادی - استخراج و حل معادله حاکم بر حرکت با وجود میراگر اصطکاکی	
۷	ارتعاشات آزاد یک درجه آزادی - روش انرژی، روش رابلی، جرم موثر، سفتی موثر	
۸	ارتعاشات اجباری یک درجه آزادی - مسئله تحریک با نیروی هارمونیک	
۹	ارتعاشات اجباری یک درجه آزادی - مسئله تحریک جرم دوار نامیزان، محور دوار با دیسک خارج مرکز	
۱۰	ارتعاشات اجباری یک درجه آزادی - مسئله تحریک پایه، ابزارهای سنجش ارتعاش و لرزه	
۱۱	ارتعاشات اجباری یک درجه آزادی - تحریک ضربه ای، تحریک دلخواه	
۱۲	ارتعاشات اجباری یک درجه آزادی - میرایی معادل، میراگر سازه ای	
۱۳	ارتعاشات آزاد سیستم دو درجه آزادی نامیرا - استخراج معادله، خصوصیات ماتریس ها، انواع کوپلینگ	
۱۴	ارتعاشات آزاد سیستم دو درجه آزادی نامیرا - حل مسئله مقدار ویژه، تعامد	
۱۵	ارتعاشات اجباری سیستم دو درجه آزادی نامیرا - حل معادله فرکانسی، ترسیم و تحلیل نمودارها	
۱۶	ارتعاشات اجباری سیستم دو درجه آزادی نامیرا - جاذب ارتعاشی	

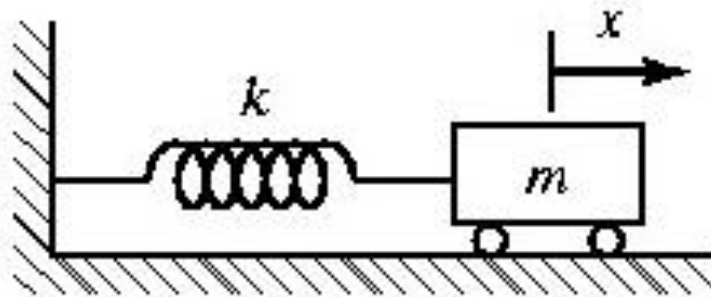
Basic Concepts of Vibration

- Degree of Freedom (*d.o.f.*) = min. no. of independent coordinates required to determine completely the positions of all parts of a system at any instant of time
- Examples of **single** degree-of-freedom systems:

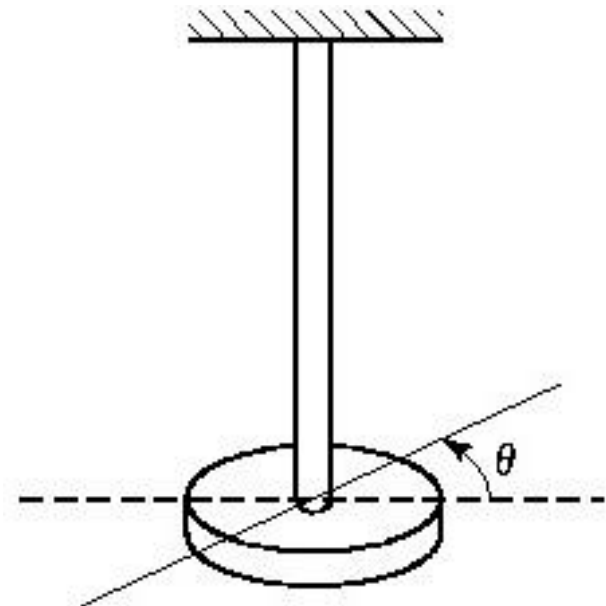


Basic Concepts of Vibration

- Examples of **single** degree-of-freedom systems:



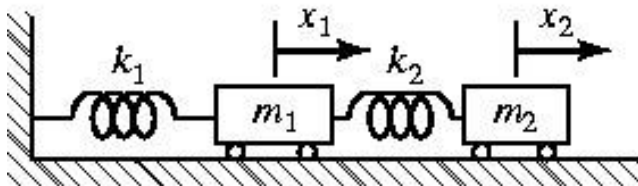
(b) Spring-mass system



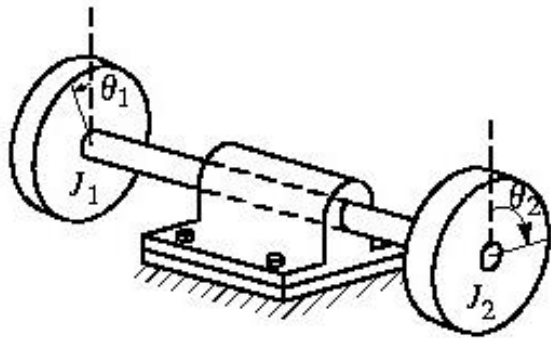
(c) Torsional system

Basic Concepts of Vibration

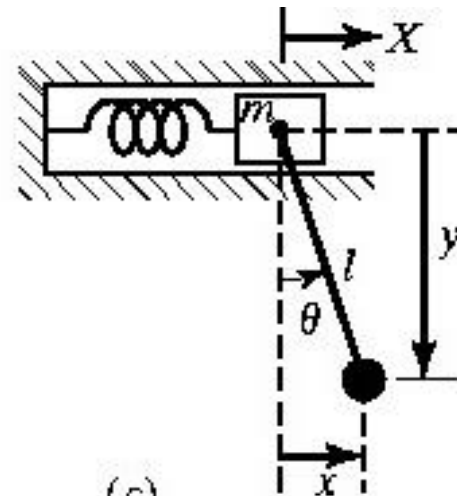
- Examples of **Two** degree-of-freedom systems:



(a)



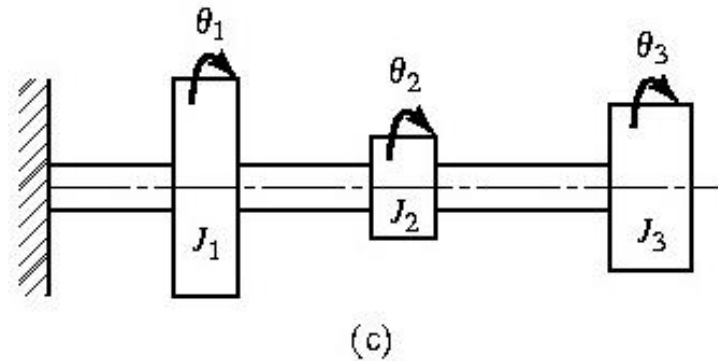
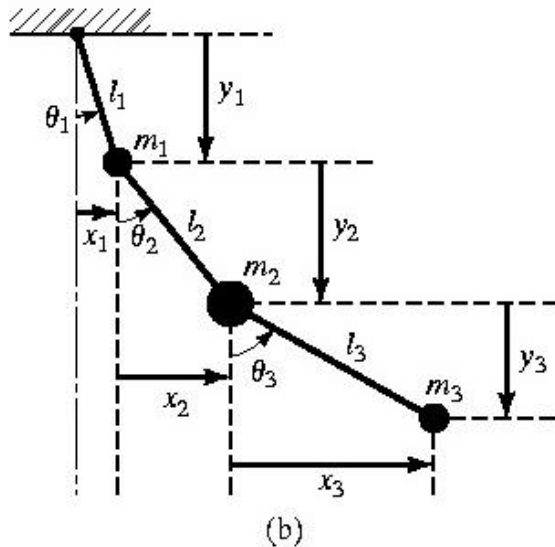
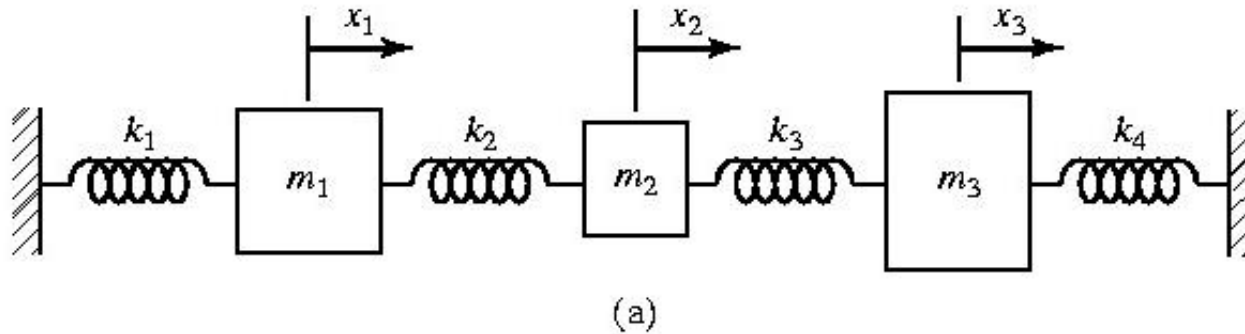
(b)



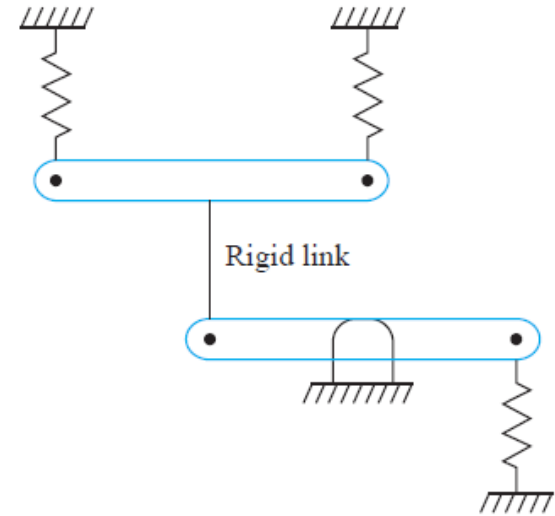
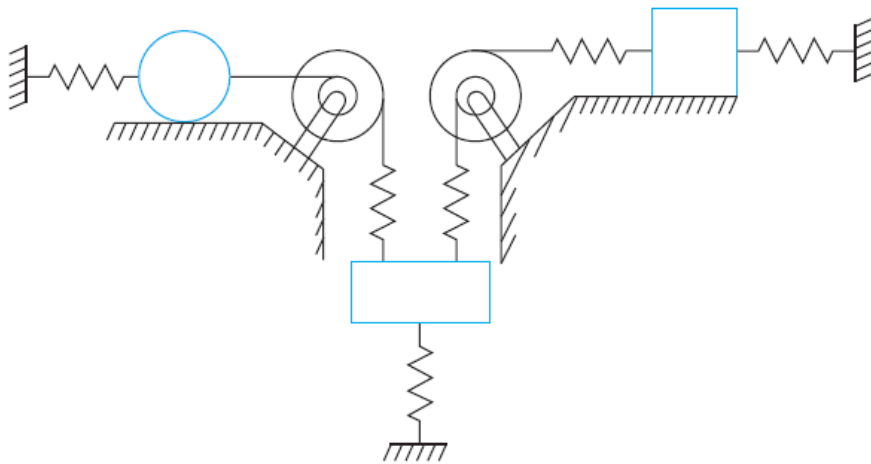
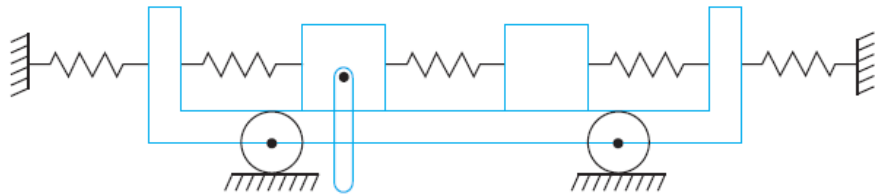
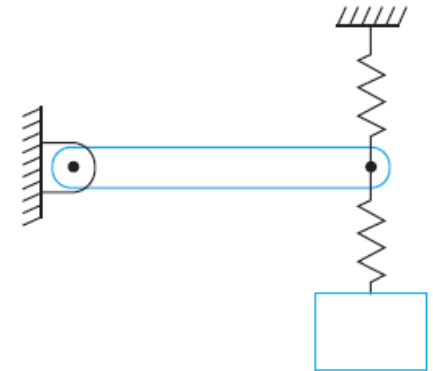
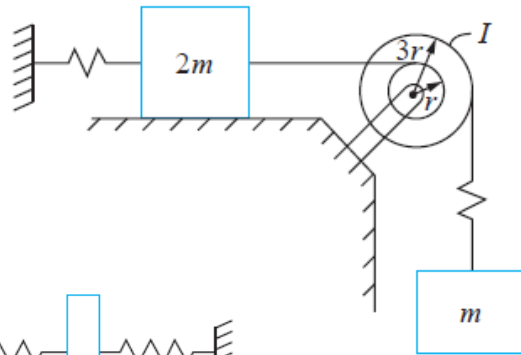
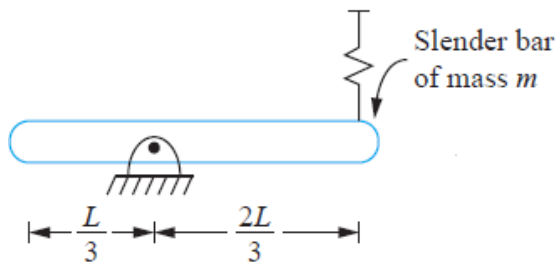
(c)

Basic Concepts of Vibration

- Examples of **Three** degree of freedom systems:

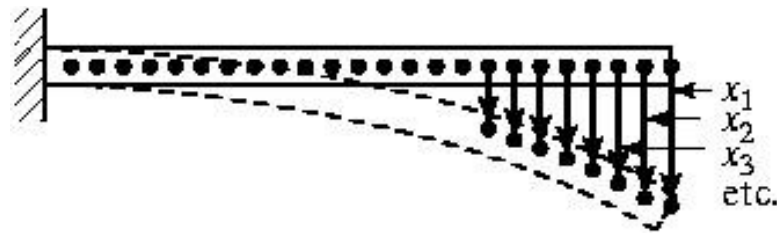


What are their DOFs?



Basic Concepts of Vibration

- Example of **Infinite**-number-of-degrees-of-freedom system:



- *Infinite* number of degrees of freedom system are termed **continuous** or **distributed** systems
- *Finite* number of degrees of freedom are termed **discrete** or **lumped** parameter systems
- More accurate results obtained by **increasing** number of degrees of freedom

Classification of Vibration

- ❑ **Free** Vibration:

A system is left to vibrate on its own after an initial disturbance and no external force acts on the system. E.g. simple pendulum

- ❑ **Forced** Vibration:

A system that is subjected to a repeating external force. E.g. oscillation arises from diesel engines

- *Resonance* occurs when the frequency of the external force coincides with one of the natural frequencies of the system

Classification of Vibration

- **Undamped** Vibration:

When ***no*** energy is lost or dissipated in friction or other resistance during oscillations

- **Damped** Vibration:

When ***any*** energy is lost or dissipated in friction or other resistance during oscillations

- **Linear** Vibration:

When ***all*** basic components of a vibratory system, i.e. the spring, the mass and the damper behave linearly

Classification of Vibration

- ❑ **Nonlinear** Vibration:

If ***any*** of the components behave nonlinearly

- ❑ **Deterministic** Vibration:

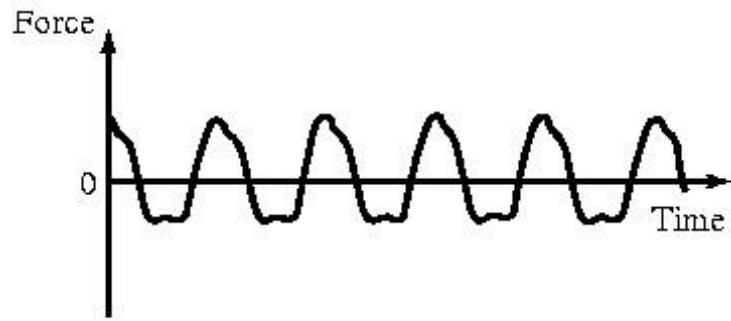
If the value or magnitude of the excitation (force or motion) acting on a vibratory system is known at any given time

- ❑ **Nondeterministic or random** Vibration:

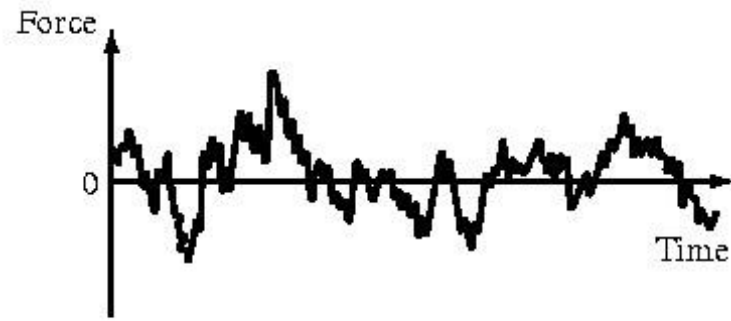
When the value of the excitation at a given time cannot be predicted

Classification of Vibration

- Examples of deterministic and random excitation:



(a) A deterministic (periodic) excitation



(b) A random excitation

Vibration Analysis Procedure

Step 1: Mathematical Modeling

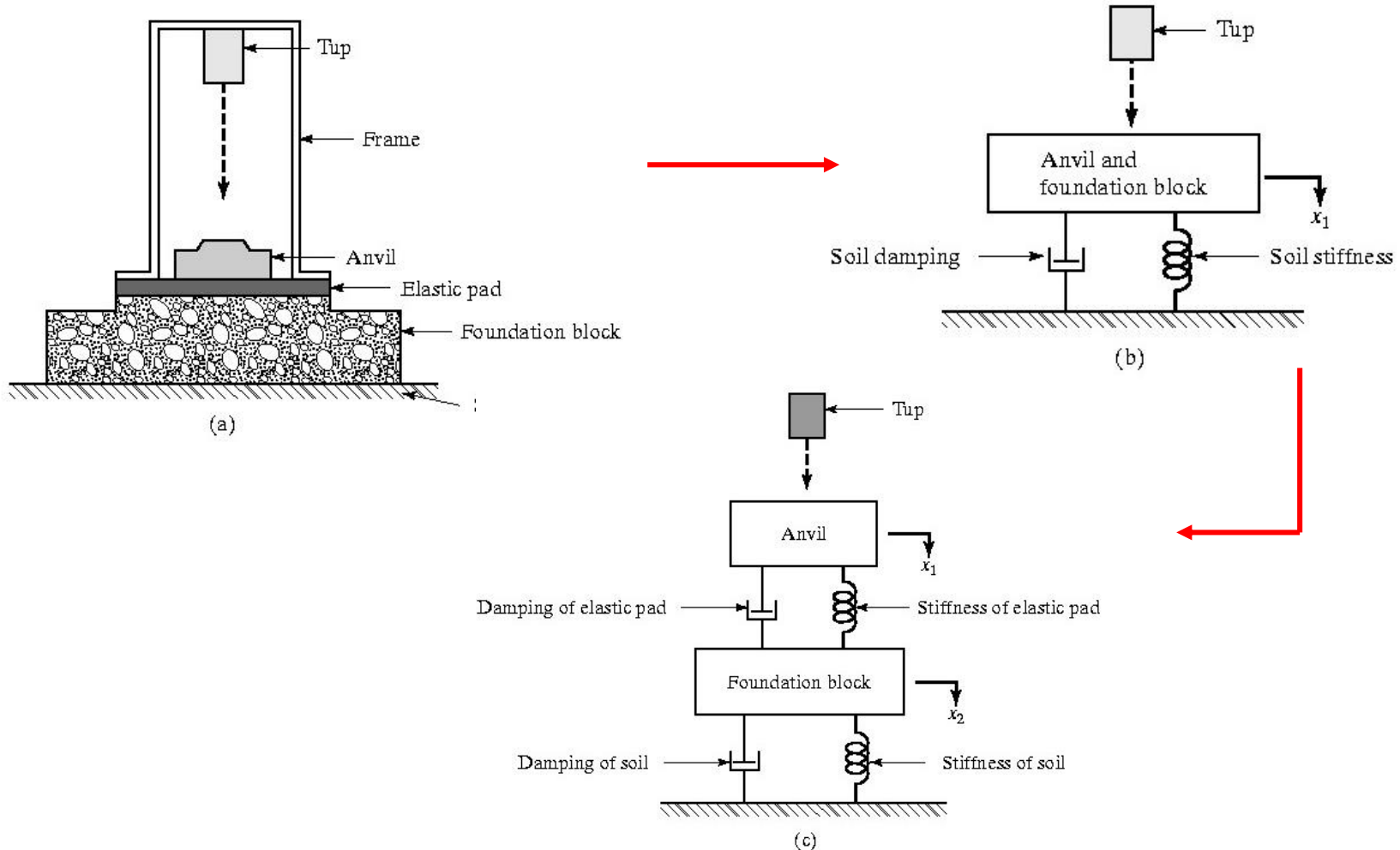
Step 2: Derivation of Governing Equations

Step 3: Solution of the Governing Equations

Step 4: Interpretation of the Results

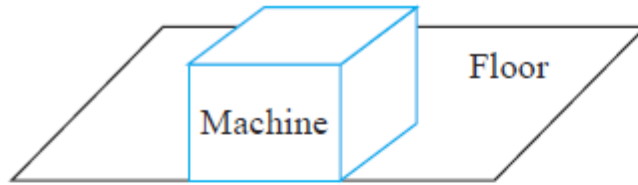
Equivalent model of systems

□ Example of the modeling of a forging hammer:



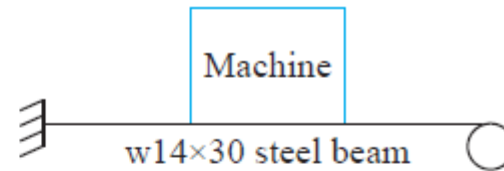
Equivalent model of systems

Example 1:



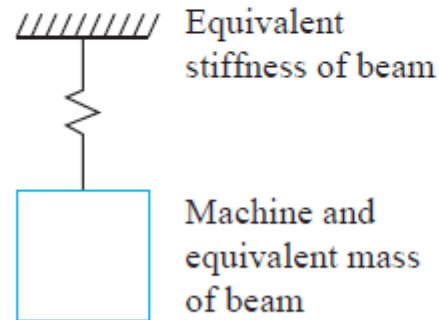
(a)

Example 2:



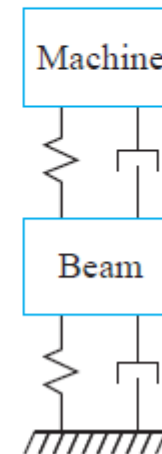
(b)

SDOF
DOF=1



(c)

MDOF
DOF=2



(d)

Equivalent model of systems

MDOF

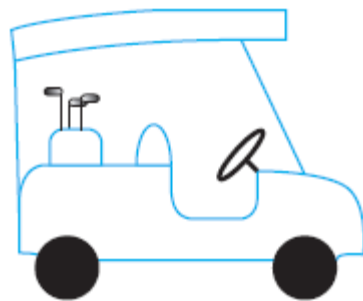
Example 3:

DOF= 3 if body 1 has no rotation

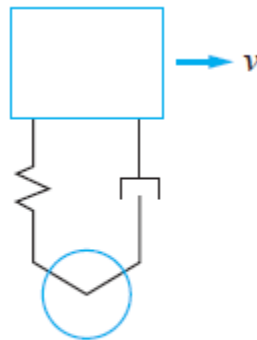
DOF= 4 if body 1 has rotation

SDOF

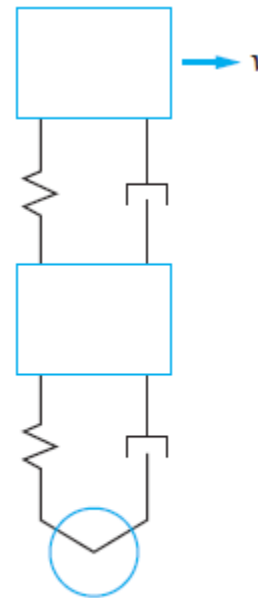
DOF=2



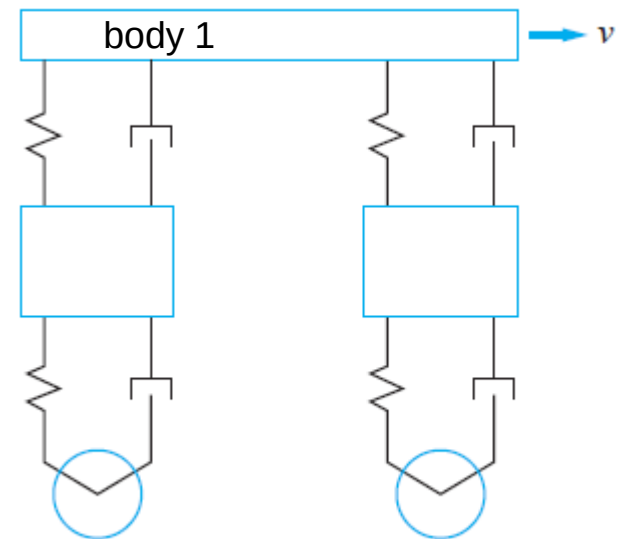
(a)



(b)



(c)



(d)

Spring Elements

- *Linear* spring is a type of mechanical link that is generally assumed to have negligible mass and damping
- *Spring force* is given by:

$$F = kx \quad (1.1)$$

F = spring force,

k = spring stiffness or spring constant, and

x = deformation (displacement of one end with respect to the other)

Spring Elements

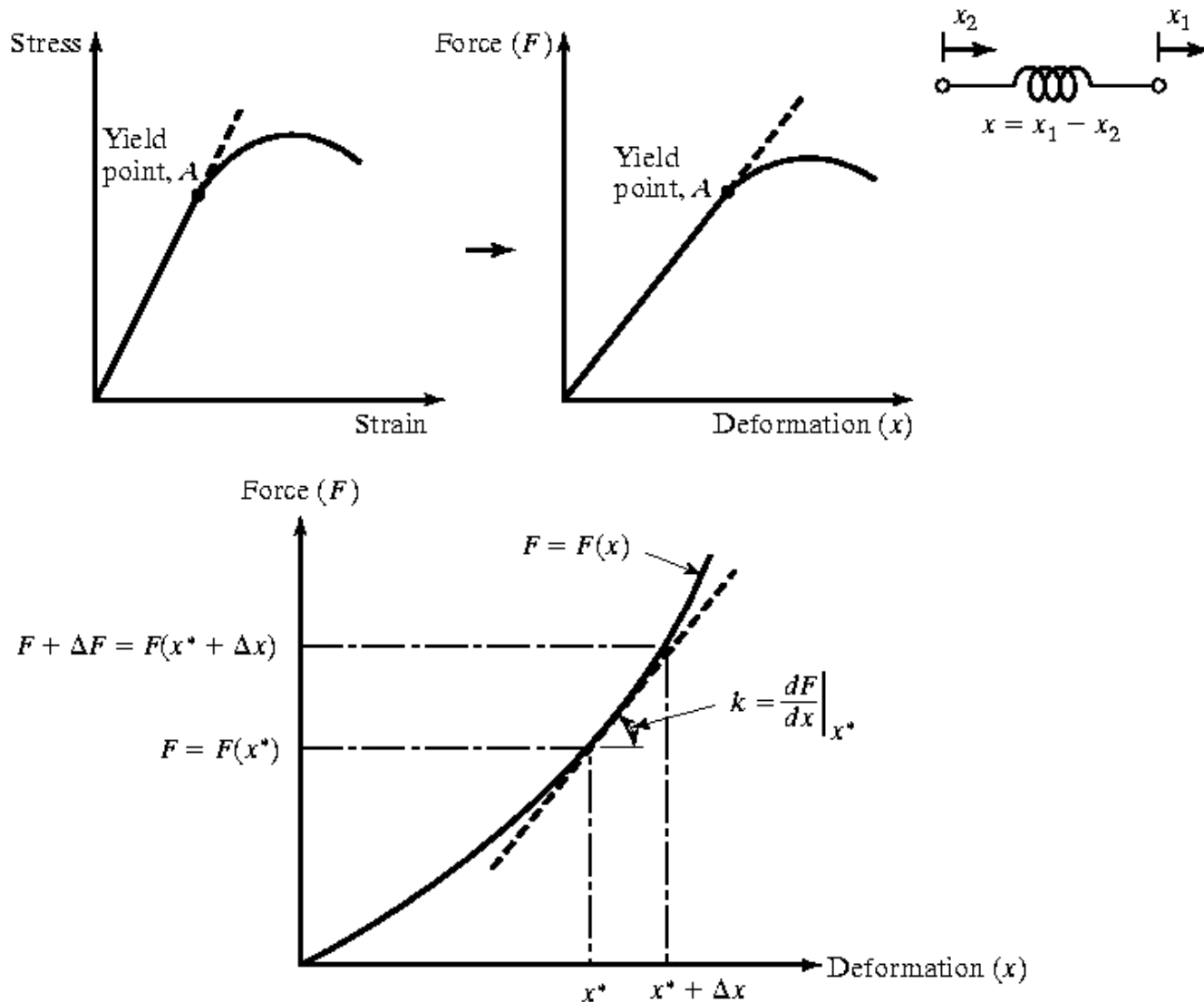
- *Work done (U)* in deforming a spring or the strain (potential) energy is given by:

$$U = \frac{1}{2} kx^2 \quad (1.2)$$

- When an incremental force ΔF is added to F :

$$\begin{aligned} F + \Delta F &= F(x^* + \Delta x) \\ &= F(x^*) + \left. \frac{dF}{dx} \right|_{x^*} (\Delta x) \\ &\quad + \frac{1}{2!} \left. \frac{d^2 F}{dx^2} \right|_{x^*} (\Delta x)^2 + \dots \end{aligned} \quad (1.3)$$

Spring Elements



Spring Elements

- *Static deflection* of a beam at the free end is given by:

$$\delta_{st} = \frac{Wl^3}{3EI} \quad (1.6)$$

$W = mg$ is the weight of the mass m ,

E = Young's Modulus, and

I = moment of inertia of cross-section of beam

- *Spring Constant* is given by:

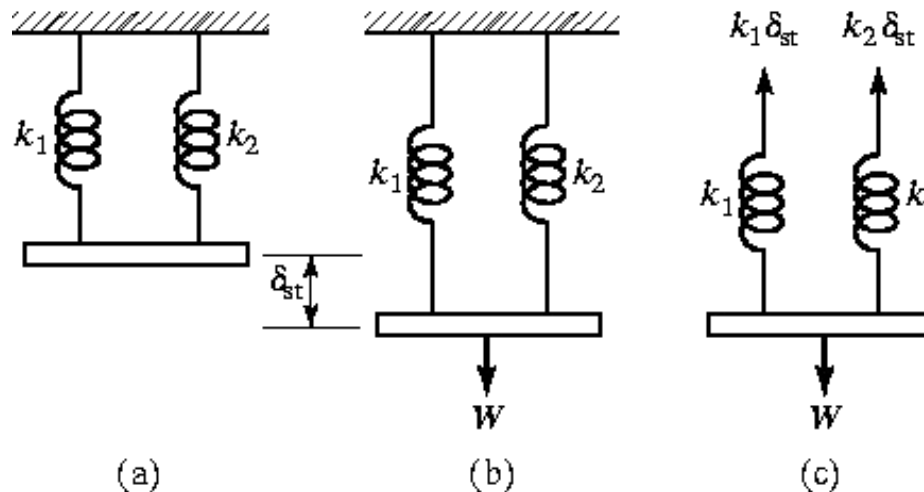
$$k = \frac{W}{\delta_{st}} = \frac{3EI}{l^3} \quad (1.7)$$

Spring Elements

□ Combination of Springs:

1) *Springs in **parallel*** – if we have n spring constants $k_1, k_2, \dots, k_n$ in *parallel*, then the equivalent spring constant k_{eq} is:

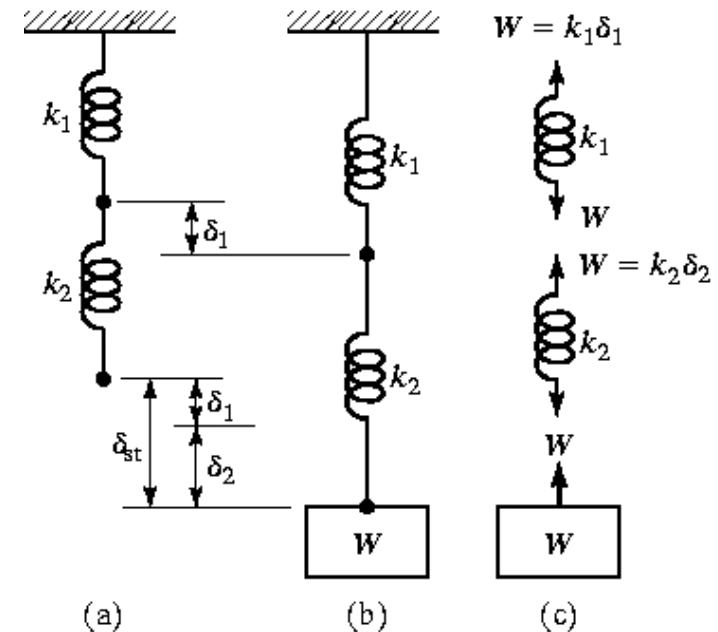
$$k_{eq} = k_1 + k_2 + \dots + k_n \quad (1.11)$$



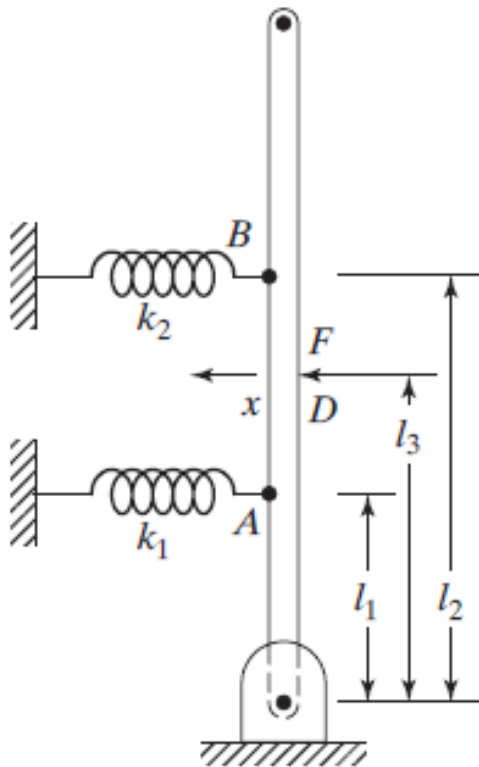
Spring Elements

- Combination of Springs:
2) *Springs in series* – if we have n spring constants $k_1, k_2, \dots, k_n$ in *series*, then the equivalent spring constant k_{eq} is:

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \dots + \frac{1}{k_n} \quad (1.17)$$



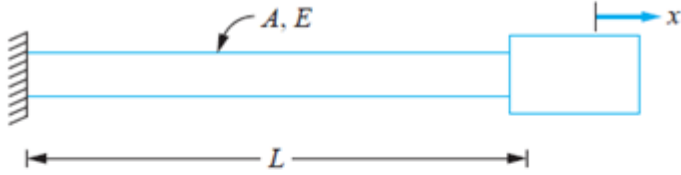
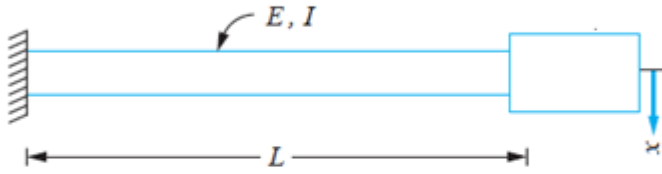
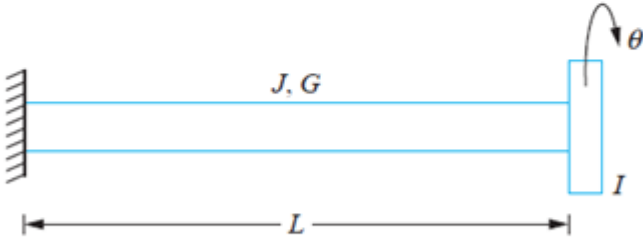
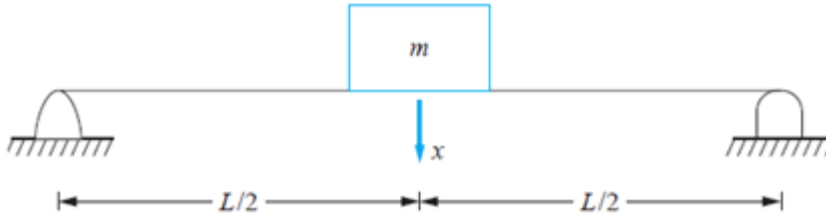
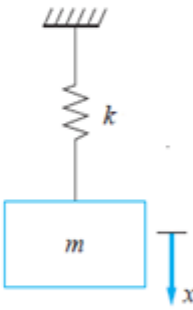
Equivalent Stiffness



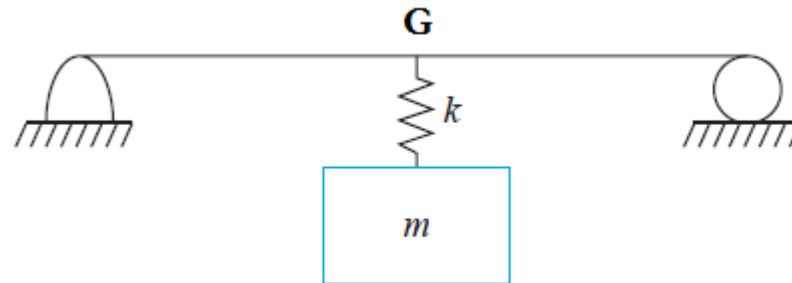
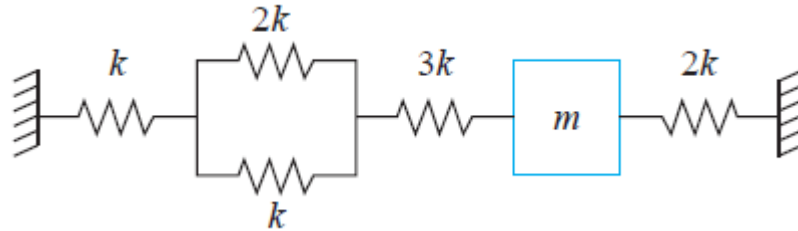
$$U_{main} = U_e \rightarrow \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 x_2^2 = \frac{1}{2} k_e x^2$$

$$\rightarrow k_e = k_1 \frac{x_1^2}{x^2} + k_2 \frac{x_2^2}{x^2} \rightarrow k_e = k_1 \left(\frac{L_1}{L_3} \right)^2 + k_2 \left(\frac{L_2}{L_3} \right)^2$$

Elastic elements as springs

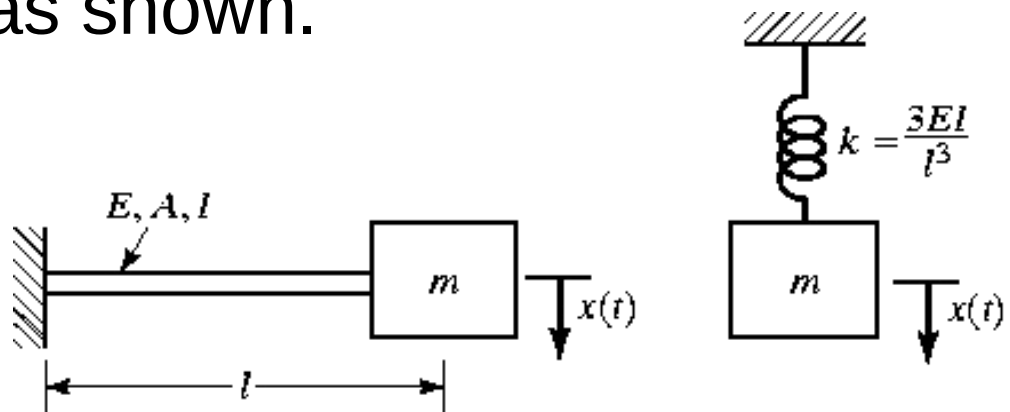
System	Stiffness Coeff.	SDOF Model
	$k = \frac{AE}{L}$	
	$k = \frac{3EI}{L^3}$	
	$k = \frac{JG}{L}$	
	$k = \frac{48EI}{L^3}$	

What are the equivalent stiffnesses?



Mass or Inertia Elements

- Using mathematical model to represent the actual vibrating system
 - E.g. In figure below, the mass and damping of the beam can be disregarded; the system can thus be modeled as a spring-mass system as shown.



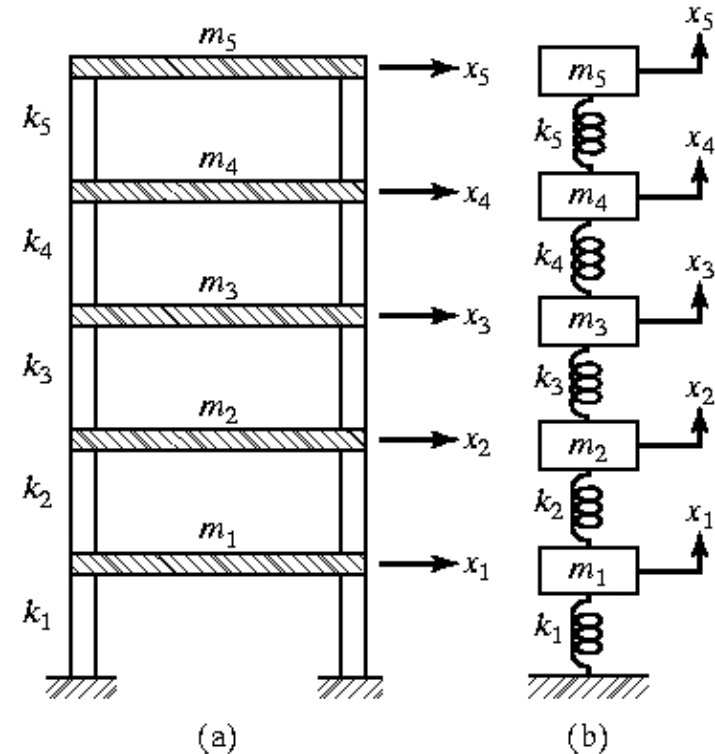
(a) Actual system

(b) Single degree of freedom model

Mass or Inertia Elements

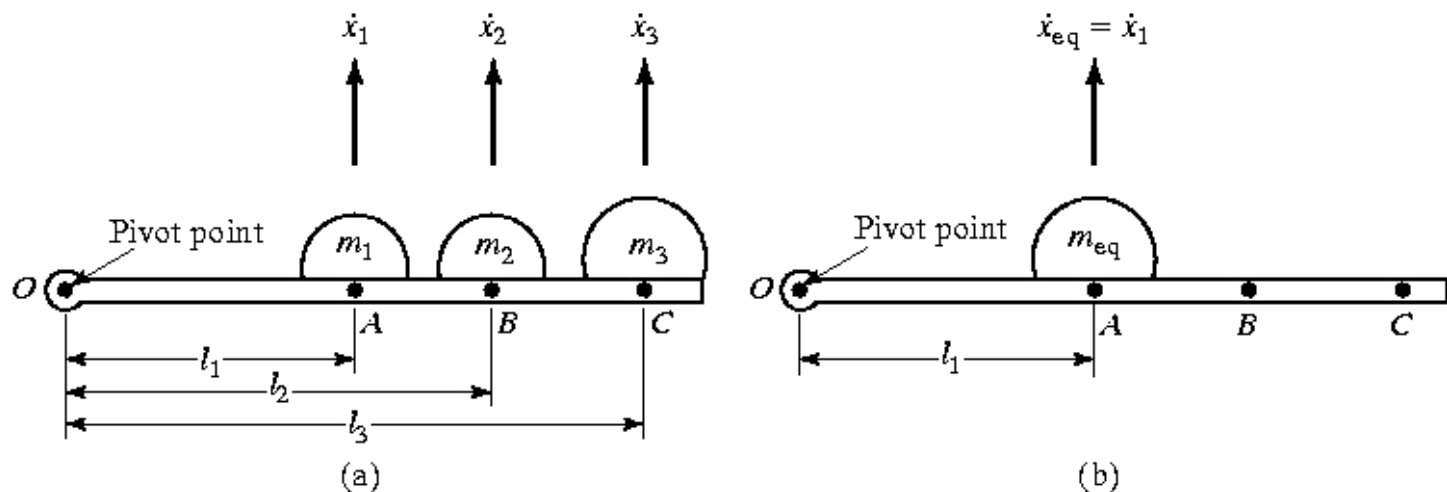
□ Combination of Masses

- E.g. Assume that the mass of the frame is negligible compared to the masses of the floors. The masses of various floor levels represent the mass elements, and the elasticities of the vertical members denote the spring elements.



Mass or Inertia Elements

□ Case 1: Translational Masses Connected by a Rigid Bar



➤ Velocities of masses can be expressed as:

$$\dot{x}_2 = \frac{l_2}{l_1} \dot{x}_1 \quad \dot{x}_3 = \frac{l_3}{l_1} \dot{x}_1 \quad (1.18)$$

- **Case 1:** Translational Masses Connected by a Rigid Bar

$$\dot{x}_{eq} = \dot{x}_1 \quad (1.19)$$

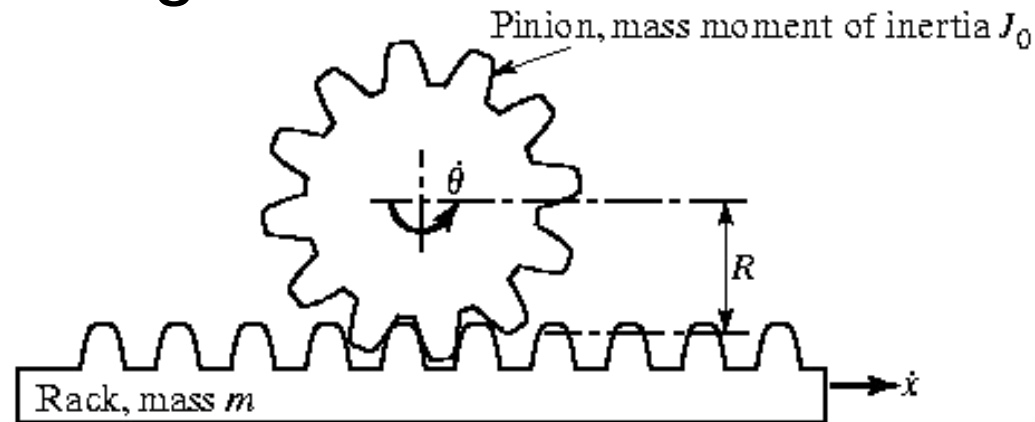
- By equating the kinetic energy of the system:

$$\frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + \frac{1}{2}m_3\dot{x}_3^2 = \frac{1}{2}m_{eq}\dot{x}_{eq}^2 \quad (1.20)$$

$$m_{eq} = m_1 + \left(\frac{l_2}{l_1}\right)^2 m_2 + \left(\frac{l_3}{l_1}\right)^2 m_3 \quad (1.21)$$

Mass or Inertia Elements

□ Case 2: Translational and Rotational Masses Coupled Together



m_{eq} = single equivalent translational mass

$\dot{x}$ = translational velocity

$\dot{\theta}$ = rotational velocity

J_0 = mass moment of inertia

J_{eq} = single equivalent rotational mass

□ Case 2: Translational and Rotational Masses Coupled Together

1. Equivalent translational mass:

- Kinetic energy of the two masses is given by:

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_0\dot{\theta}^2 \quad (1.22)$$

- Kinetic energy of the equivalent mass is given by:

$$T_{eq} = \frac{1}{2}m_{eq}\dot{x}_{eq}^2 \quad (1.23)$$

□ Case 2: Translational and Rotational Masses Coupled Together

➤ Since $\dot{\theta} = \frac{\dot{x}}{R}$ and $\dot{x}_{eq} = \dot{x}$, equating T_{eq} & T gives

$$m_{eq} = m + \frac{J_0}{R^2} \quad (1.24)$$

2. Equivalent rotational mass:

➤ Here, $\dot{\theta}_{eq} = \dot{\theta}$ and $\dot{x} = \dot{\theta}R$, equating T_{eq} and T gives

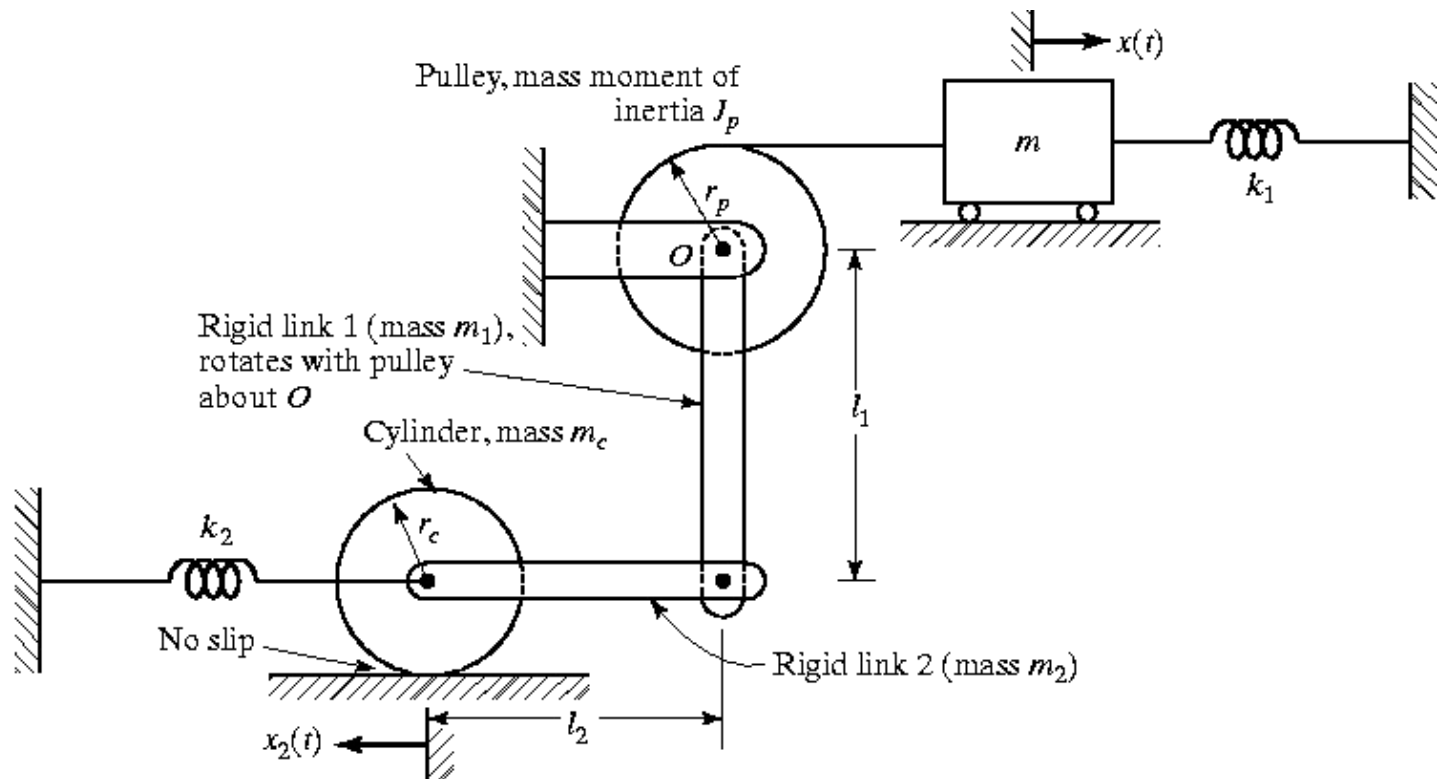
$$\frac{1}{2} J_{eq} \dot{\theta}^2 = \frac{1}{2} m (\dot{\theta}R)^2 + \frac{1}{2} J_0 \dot{\theta}^2$$

$$\text{or} \quad J_{eq} = J_0 + mR^2 \quad (1.25)$$

Example 1.6

Equivalent Mass of a System

Find the equivalent mass of the system shown in Fig. 1.31, where the rigid link 1 is attached to the pulley and rotates with it.



Example 1.6 Solution

$$\begin{aligned} T = & \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J_p \left(\frac{\dot{x}}{r_p} \right)^2 + \frac{1}{2} \left(\frac{m l_1^2}{3} \right) \left(\frac{\dot{x}}{r_p} \right)^2 + \frac{1}{2} m_2 \left(\frac{\dot{x} l_1}{r_p} \right)^2 \\ & + \frac{1}{2} \left(\frac{m_c r_c^2}{2} \right) \left(\frac{\dot{x} l_1}{r_p r_c} \right)^2 + \frac{1}{2} m_c \left(\frac{\dot{x} l_1}{r_p} \right)^2 \end{aligned} \quad (\text{E.2})$$

By equating Equation (E.2) to the kinetic energy of the equivalent system

$$T = \frac{1}{2} m_{eq} \dot{x}^2 \quad (\text{E.3})$$

Example 1.6 Solution

we obtain the equivalent mass of the system as

$$m_{eq} = m + \frac{J_p}{r_p^2} + \frac{1}{3} \frac{m_1 l_1^2}{r_p^2} + \frac{m_2 l_1^2}{r_p^2} + \frac{1}{2} \frac{m_c l_1^2}{r_p^2} + m_c \frac{l_1^2}{r_p^2} \quad (\text{E.4})$$

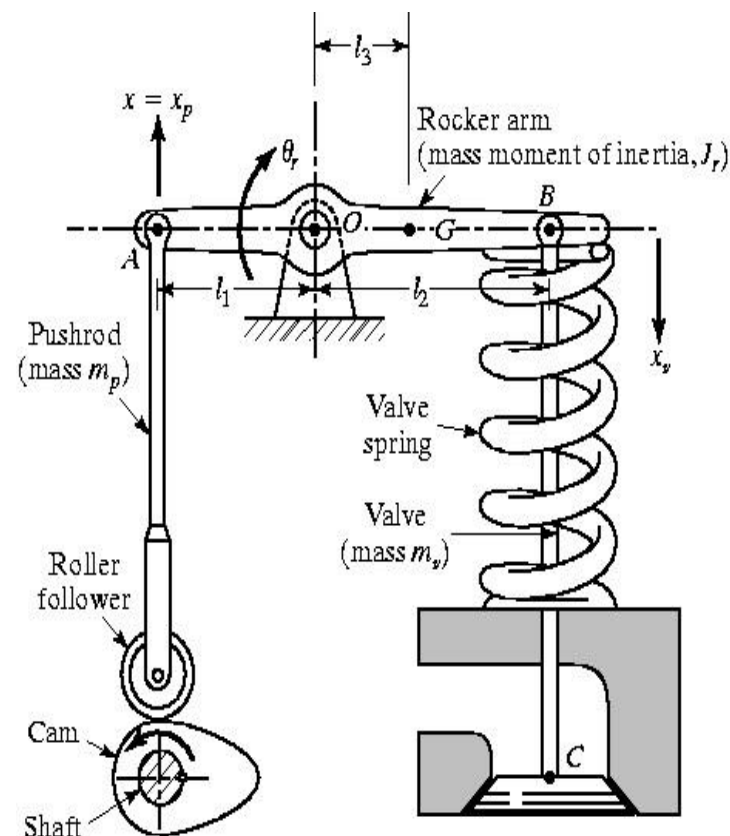
Example 1.7

Cam-Follower Mechanism

A cam-follower mechanism is used to convert the rotary motion of a shaft into the oscillating or reciprocating motion of a valve.

The follower system consists of a pushrod of mass m_p , a rocker arm of mass m_r , and mass moment of inertia J_r about its C.G., a valve of mass m_v , and a valve spring of negligible mass.

Find the equivalent mass (m_{eq}) of this cam-follower system by assuming the location of m_{eq} as (i) pt A and (ii) pt C.



Example 1.7 Solution

The kinetic energy of the system (T) is:

$$T = \frac{1}{2}m_p\dot{x}_p^2 + \frac{1}{2}m_v\dot{x}_v^2 + \frac{1}{2}J_r\dot{\theta}_r^2 + \frac{1}{2}m_r\dot{x}_r^2 \quad (\text{E.1})$$

If m_{eq} denotes equivalent mass placed at pt A , with $\dot{x}_{\text{eq}} = \dot{x}$, the kinetic energy equivalent mass system T_{eq} is:

$$T_{\text{eq}} = \frac{1}{2}m_{\text{eq}}\dot{x}_{\text{eq}}^2 \quad (\text{E.2})$$

Example 1.7 Solution

By equating T and T_{eq} , and note that

$$\dot{x}_p = \dot{x}, \quad \dot{x}_v = \frac{\dot{x}l_2}{l_1}, \quad \dot{x}_r = \frac{\dot{x}l_3}{l_1}, \quad \text{and} \quad \dot{\theta}_r = \frac{\dot{x}}{l_1}$$
$$m_{\text{eq}} = m_p + \frac{J_r}{l_1^2} + m_v \frac{l_2^2}{l_1^2} + m_r \frac{l_3^2}{l_1^2} \quad (\text{E.3})$$

Similarly, if equivalent mass is located at point C, $\dot{x}_{\text{eq}} = \dot{x}_v$, hence,

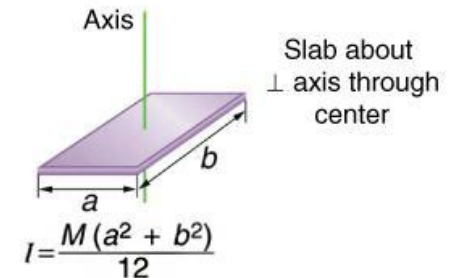
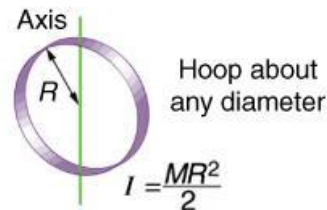
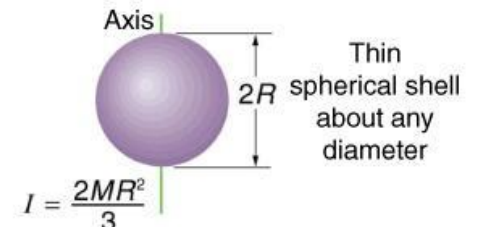
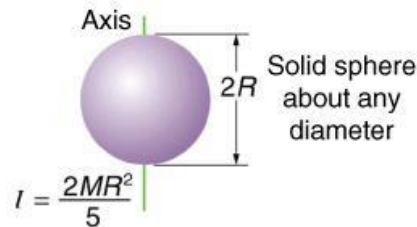
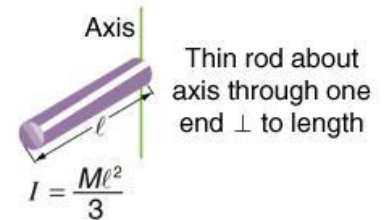
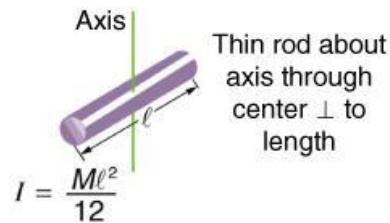
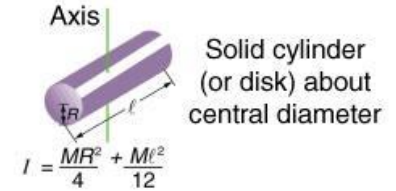
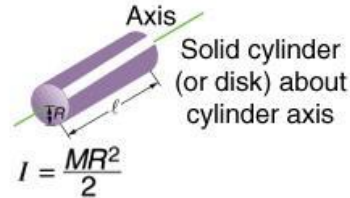
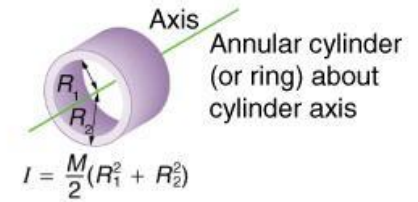
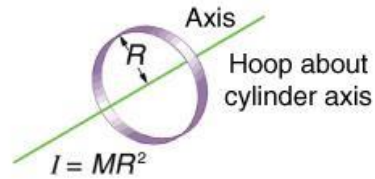
$$T_{\text{eq}} = \frac{1}{2} m_{\text{eq}} \dot{x}_{\text{eq}}^2 = \frac{1}{2} m_{\text{eq}} \dot{x}_v^2 \quad (\text{E.4})$$

Example 1.7 Solution

Equating (E.4) and (E.1) gives

$$m_{\text{eq}} = m_v + \frac{J_r}{l_2^2} + m_p \left(\frac{l_1}{l_2} \right)^2 + m_r \left(\frac{l_3^2}{l_1^2} \right)^2 \quad (\text{E.5})$$

Moment of Inertia

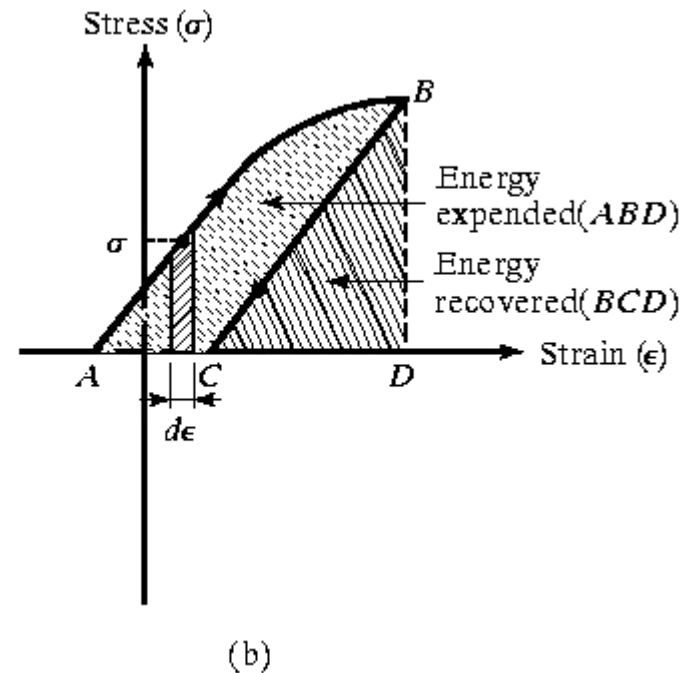
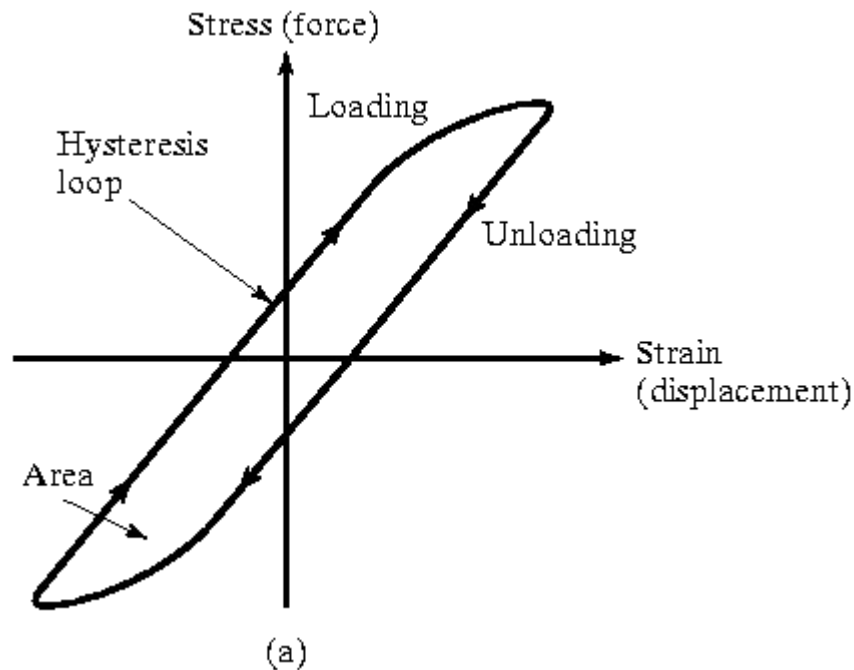


Damping Elements

- ❑ **Viscous** Damping:
Damping force is proportional to the velocity of the vibrating body in a fluid medium such as air, water, gas, and oil.
- ❑ **Coulomb** or **Dry Friction** Damping:
Damping force is constant in magnitude but opposite in direction to that of the motion of the vibrating body between dry surfaces
- ❑ **Material** or **Solid** or **Hysteretic** Damping:
Energy is absorbed or dissipated by material during deformation due to friction between internal planes

Damping Elements

Hysteresis loop for elastic materials:



Damping Elements

- *Shear Stress (τ)* developed in the fluid layer at a distance y from the fixed plate is:

$$\tau = \mu \frac{du}{dy} \quad (1.26)$$

where $du/dy = v/h$ is the velocity gradient.

- *Shear or Resisting Force (F)* developed at the bottom surface of the moving plate is:

$$F = \tau A = \mu \frac{Av}{h} = cv \quad (1.27)$$

where A is the surface area of the moving plate.

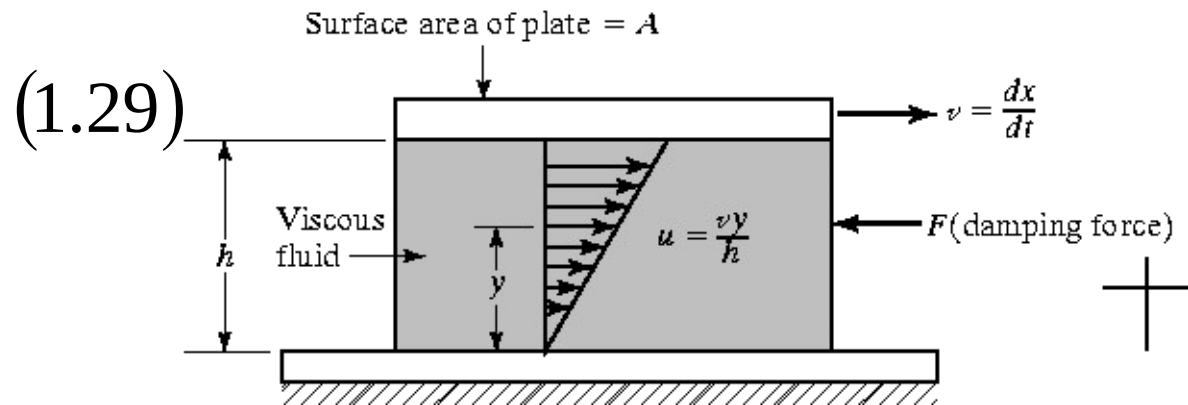
Damping Elements

and
$$c = \frac{\mu A}{h} \quad (1.28)$$

is called the damping constant.

- If a damper is **nonlinear**, a linearization process is used about the operating velocity (v^*) and the equivalent damping constant is:

$$c = \left. \frac{dF}{dv} \right|_{v^*}$$



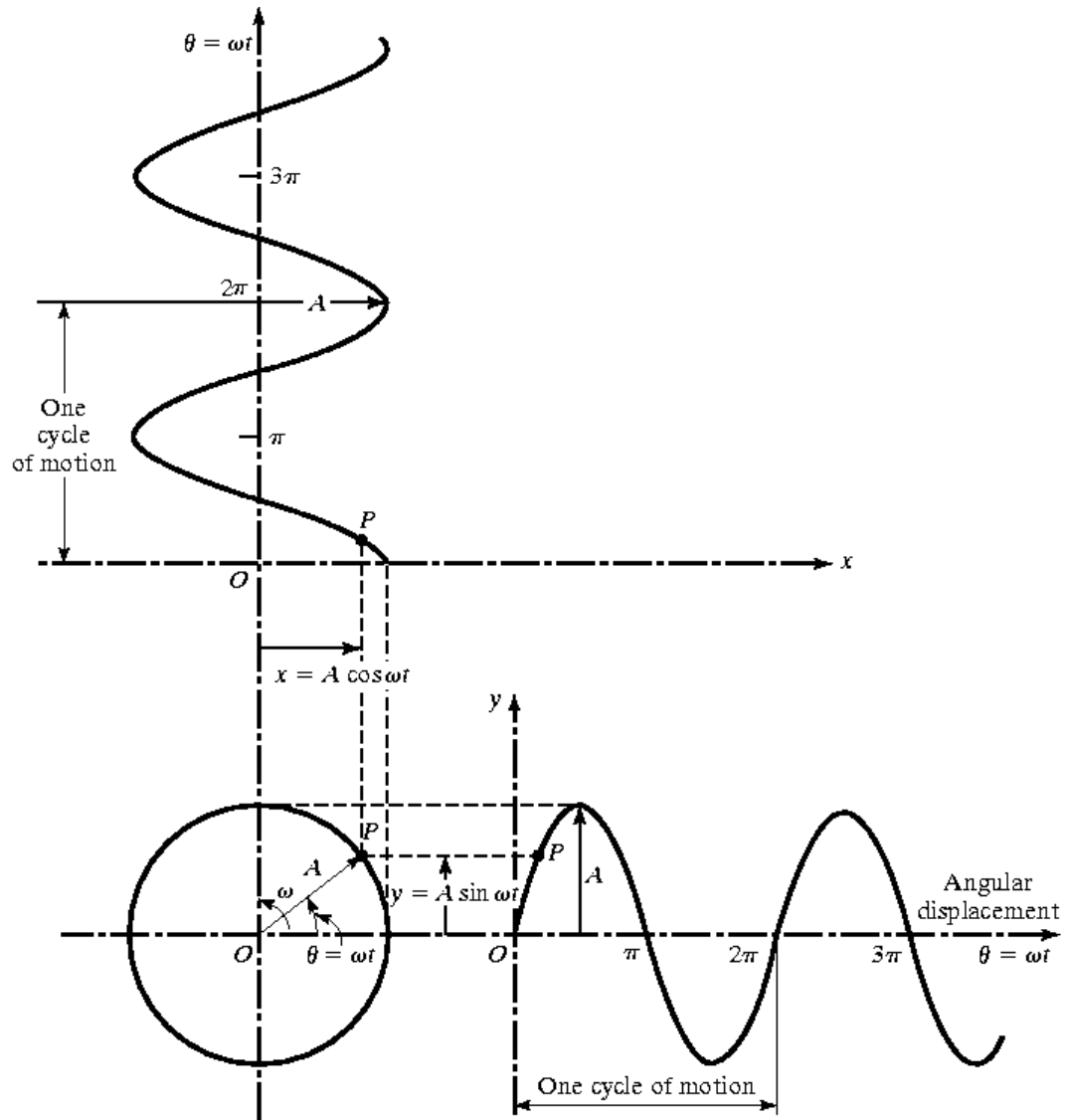
Harmonic Motion

- ❑ **Periodic** Motion: motion repeated after equal intervals of time
- ❑ **Harmonic** Motion: simplest type of periodic motion $x = A \sin \theta = A \sin \omega t$ (1.30)
- ❑ Displacement (x): *(on horizontal axis)*
- ❑ Velocity: $\frac{dx}{dt} = \omega A \cos \omega t$ (1.31)
- ❑ Acceleration:

$$\frac{d^2 x}{dt^2} = -\omega^2 A \sin \omega t = -\omega^2 x \quad (1.32)$$

Harmonic Motion

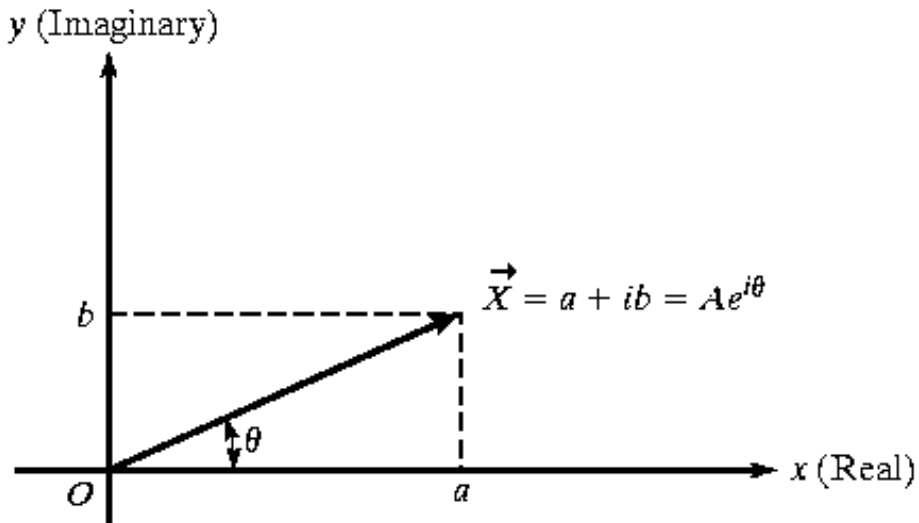
- Scotch yoke mechanism:
The similarity between cyclic (harmonic) and sinusoidal motion.



Harmonic Motion

- **Complex number** representation of harmonic motion:
$$\vec{X} = a + ib \quad (1.35)$$

where $i = \sqrt{-1}$ and a and b denote the real and imaginary x and y components of X , respectively.



Harmonic Motion

- Also, Eqn. (1.36) can be expressed as

$$\vec{X} = A\cos\theta + iA\sin\theta \quad (1.36)$$

$$\vec{X} = A(\cos\theta + i\sin\theta) = Ae^{i\theta} \quad (1.43)$$

- Thus,

$$A_j = \sqrt{(a_j^2 + b_j^2)}; \quad j = 1, 2 \quad (1.47)$$

$$\theta_j = \tan^{-1}\left(\frac{b_j}{a_j}\right); \quad j = 1, 2 \quad (1.48)$$

Harmonic Motion

□ Operations on **Harmonic** Functions:

➤ Rotating Vector, $\vec{X} = Ae^{i\omega t}$ (1.51)

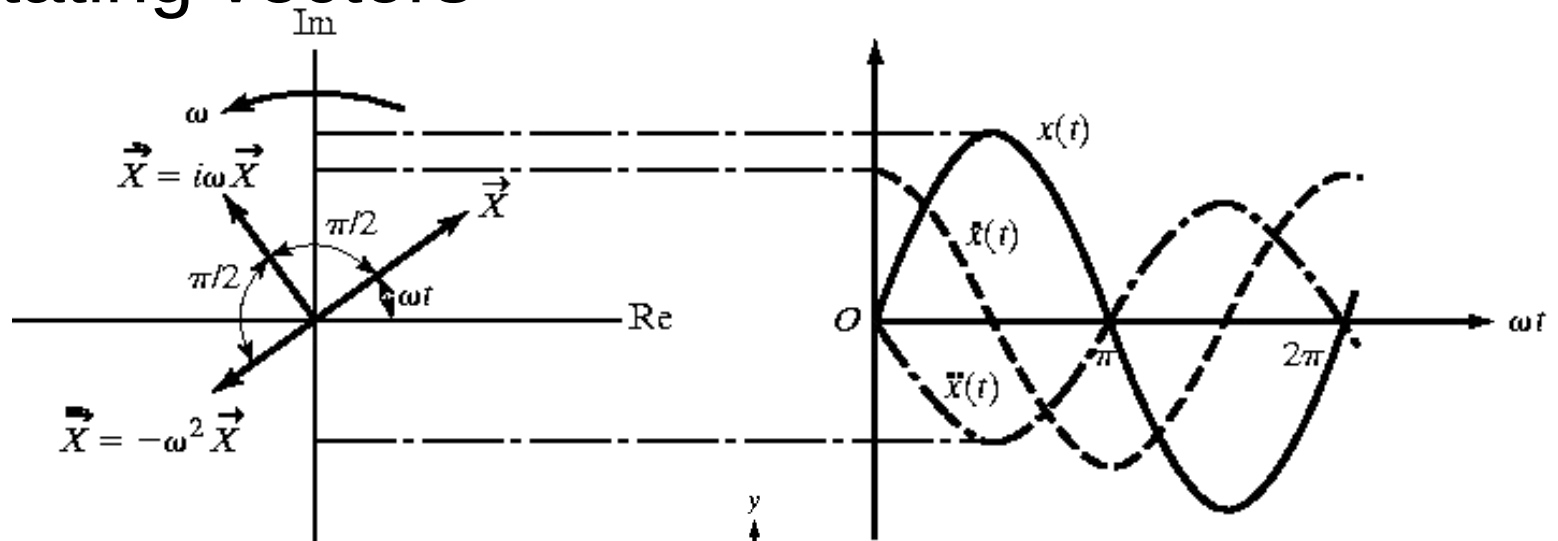
$$\text{Displacement} = \text{Re}[Ae^{i\omega t}] = A\cos \omega t \quad (1.54)$$

$$\begin{aligned} \text{Velocity} &= \text{Re}[i\omega Ae^{i\omega t}] = -\omega A\sin \omega t \\ &= -\omega A\cos(\omega t + 90^\circ) \end{aligned} \quad (1.55)$$

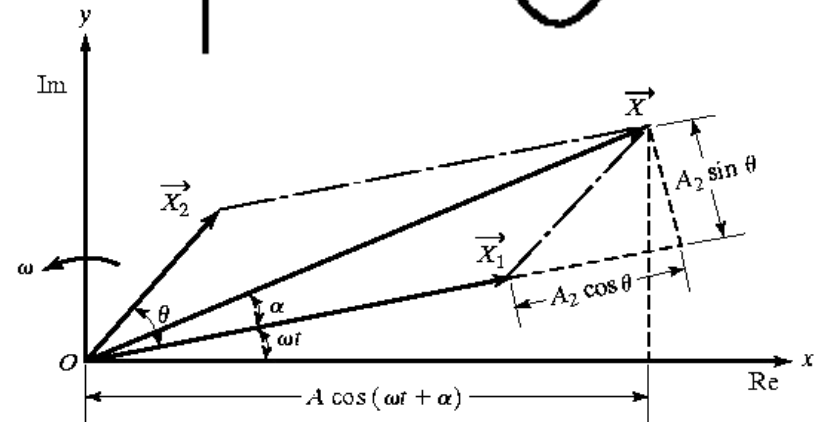
$$\begin{aligned} \text{Acceleration} &= \text{Re}[-\omega^2 Ae^{i\omega t}] \\ &= -\omega^2 A\cos \omega t \\ &= \omega^2 A\cos(\omega t + 180^\circ) \end{aligned} \quad (1.56)$$

Harmonic Motion

- Displacement, velocity, and accelerations as rotating vectors



- Vectorial addition of harmonic functions



Harmonic Motion

□ Definitions of Terminology:

- Amplitude (A) is the maximum displacement of a vibrating body from its equilibrium position
- Period of oscillation (T) is time taken to complete one cycle of motion

$$T = \frac{2\pi}{\omega} \quad (1.59)$$

- Frequency of oscillation (f) is the no. of cycles per unit time

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \quad (1.60)$$

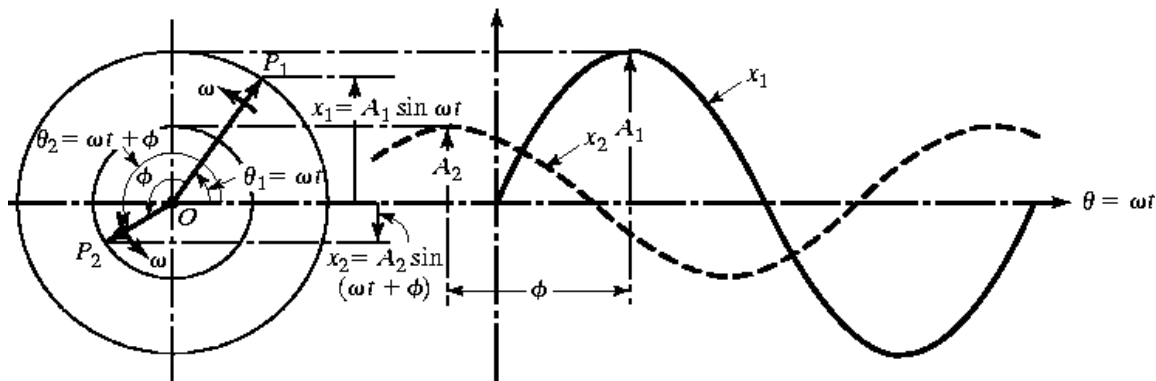
Harmonic Motion

□ Definitions of Terminology:

- Natural frequency is the frequency which a system oscillates without external forces
- Phase angle (ϕ) is the angular difference between two synchronous harmonic motions

$$x_1 = A_1 \sin \omega t \quad (1.61)$$

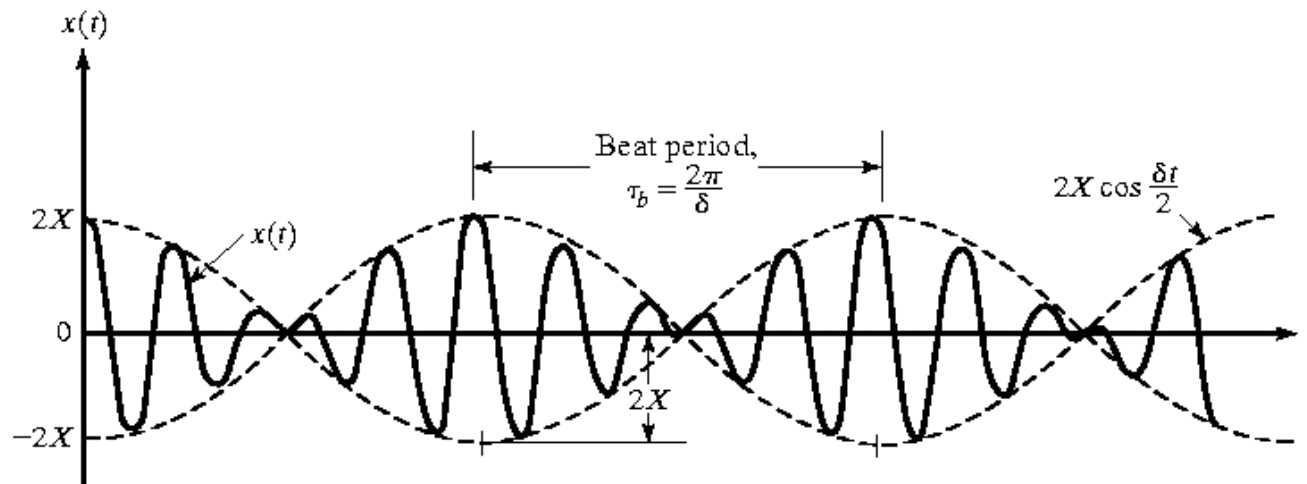
$$x_2 = A_2 \sin(\omega t + \phi) \quad (1.62)$$



Harmonic Motion

□ Definitions of Terminology:

- Beats are formed when two harmonic motions, with frequencies close to one another, are added



Harmonic Motion

□ Definitions of Terminology:

- Decibel is originally defined as a ratio of electric powers. It is now often used as a notation of various quantities such as displacement, velocity, acceleration, pressure, and power

$$\text{dB} = 10 \log \left(\frac{P}{P_0} \right) \quad (1.68)$$

$$\text{dB} = 20 \log \left(\frac{X}{X_0} \right) \quad (1.69)$$

where P_0 is some reference value of power and X_0 is specified reference voltage.

Harmonic Analysis

- Fourier Series Expansion:

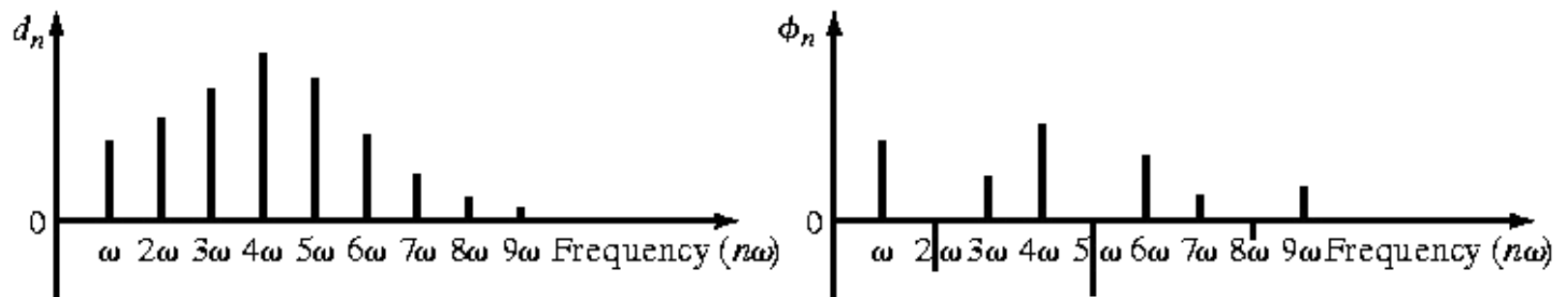
If $x(t)$ is a periodic function with period τ , its Fourier Series representation is given by

$$\begin{aligned} x(t) &= \frac{a_0}{2} + a_1 \cos \omega t + a_2 \cos 2\omega t + \dots \\ &\quad + b_1 \sin \omega t + b_2 \sin 2\omega t + \dots \\ &= \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t) \end{aligned} \quad (1.70)$$

Harmonic Analysis

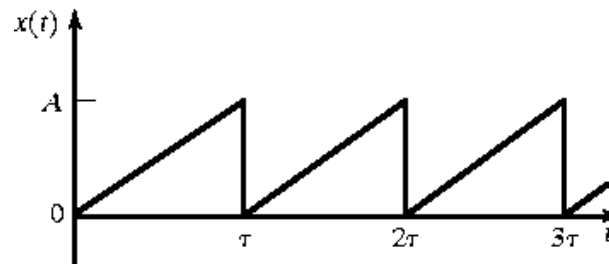
- Frequency Spectrum:

Harmonics plotted as vertical lines on a diagram of amplitude (a_n and b_n or d_n and ϕ_n) versus frequency ($n\omega$)

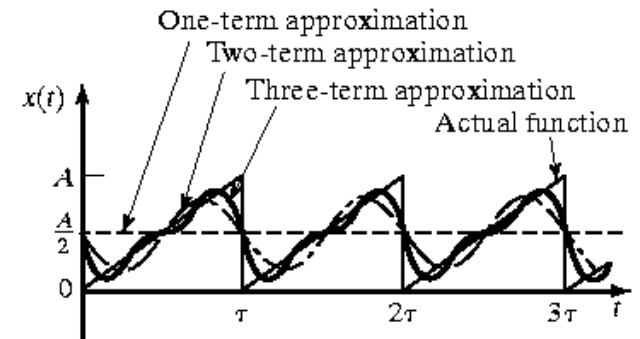


Harmonic Analysis

■ A periodic function:

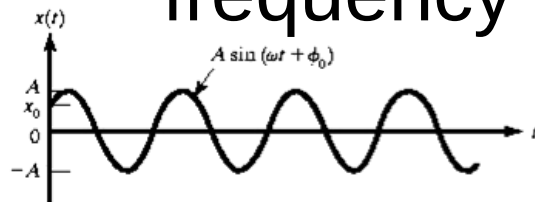


(a)

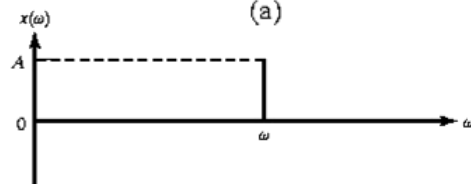


(b)

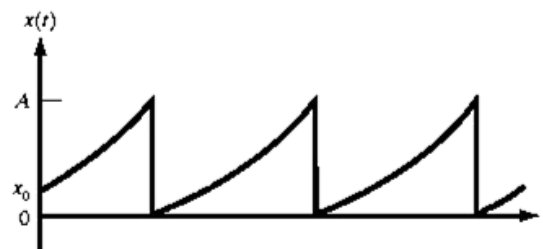
■ Representation of a function in time and frequency domain:



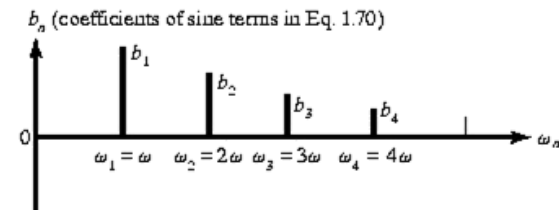
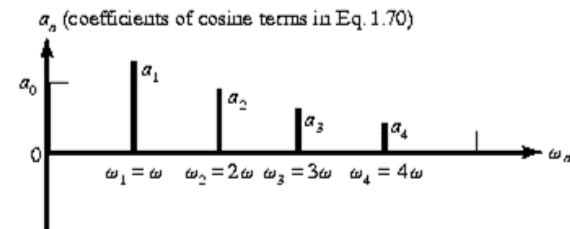
(a)



(b)



(c)



(d)